# THE ECONOMICS OF NATURAL RESOURCE USE

John M. Hartwick
*Queen's University*

Nancy D. Olewiler
*Queen's University*

1817
**HARPER & ROW, PUBLISHERS, New York**
Cambridge, Philadelphia, San Francisco,
London, Mexico City, São Paulo, Sydney

*Nature's resources are not so much an inheritance from our parents, as a loan from our children.*

*We dedicate this book to our parents and children.*

Sponsoring Editor: John Greenman
Project Editor: Mary G. Ward
Cover Design: John Hite
Text Art: Artset Ltd.
Production: Debra Forrest
Compositor: Progressive Typographers
Printer and Binder: R. R. Donnelley & Sons Company

**THE ECONOMICS OF NATURAL RESOURCE USE**



Library of Congress Cataloging in Publication Data

Hartwick, John M.
The economics of natural resource use.

Includes bibliographies.
1. Natural resources. 2. Environmental policy.
I. Olewiler, Nancy D. II. Title.
HC59.H3558 1986 333.7'13 85-14076
ISBN 0-06-042695-0

85 86 87 88 9 8 7 6 5 4 3 2 1

# CONTENTS

3. The derivation of $R$ involves summing rent per acre over all acres or integrating over a series of infinitesimal circular rings or annuluses. Before integrating,

$$R = \int_{x_0}^{x_1} 2\pi x y_1 \left(p_1 - tx - \frac{z_1 w}{y_1}\right) dx + \int_{x_1}^{x_2} 2\pi x y_2 \left(p_2 - t_2 x - \frac{z_2 w}{y_2}\right) dx$$

and $R$ in the text results after carrying out the integration operations. Below we maximize $R$ by differentiating with respect to $x_1$ and $x_2$ and set the expressions equal to zero.
4. An early statement of this was Mohring (1961). See the extensive discussion in Arnott and Stiglitz (1979).

chapter 3

# Nonrenewable Resource Use: The Theory of Depletion

## INTRODUCTION

Nonrenewable resources include energy supplies—oil, natural gas, uranium and coal—and nonenergy minerals—copper, nickel, bauxite, and zinc, to name a few. These resources are formed by geological processes that typically take millions of years, so we can view these resources for practical purposes as having a fixed *stock* of reserves. That is, there is a finite amount of the mineral in the ground, which once removed cannot be replaced.[1] Nonrenewability introduces some new problems and issues into the analysis of production from the mine or well that do not arise in the production of reproducible goods such as agricultural crops.

A mine manager must determine not only how to combine variable factor inputs such as labor and materials with fixed capital as does the farmer, but how quickly to run down the fixed stock of ore reserves through extraction of the mineral. A unit of ore extracted today means that less in total is available for tomorrow. Time plays an essential role in the analysis. Each period is different, because the stock of the resource remaining is a different size. What we are concerned with in an economic analysis of nonrenewable resources is how quickly the mineral is extracted—what the *flow* of production is over time, and when the stock will be exhausted.

In this chapter, we determine the efficient extraction path of the resource—the amount extracted in each time period. First, we examine the behavior of the individual mine operator. We then examine how a social planner would exploit the same deposit. Finally, we develop the extraction profile of a mining *industry*. In all cases, we assume that perfect competition prevails in every market. We derive the paths of mineral output, prices, and rents over time under varying assumptions about the nature of the mining process. The competitive equilibrium over time is compared to the socially optimal extraction path.

Our initial model is very simple and abstracts considerably from reality so that we can identify and examine basic concepts. The assumptions are gradually relaxed so that we can deal with increasingly complex but more realistic models. Relaxing the assumption of perfect competition is done in Chapter 4 and of certainty in Chapter 5. In examining the mine's and industry's extraction decision, we also illustrate the effects on output and prices over time of changes in particular variables affecting the mining process. What will be the effect on extraction over time of, for example, a change in extraction costs, the introduction of setup or capital costs, different qualities of ore, a change in the discount rate, the imposition of taxes?

## THE THEORY OF THE MINE

We begin with a simple model of resource extraction from an individual mine which operates in a perfectly competitive industry. The mine owner will seek to maximize the present value of profits from mineral extraction in a manner similar to that of a manager of a plant producing a reproducible good. An output level must be chosen that maximizes the difference between total revenues—the discounted value of future extractions $q_1, q_2, q_3, \ldots$ etc., multiplied by price, $p$, and total cost—the discounted value of dollars expended in getting each $q$ out of the ground. The presence of the finite stock of the mineral modifies the usual maximization condition; marginal revenue *(MR)* equals marginal cost *(MC)*, in three fundamental ways. Suppose we compare farming to copper extraction. The owner of the copper mine faces an opportunity cost not encountered by the farmer. This is the cost of using up the fixed stock at any point in time, or being left with smaller remaining reserves. To maximize profits, the operator must cover this opportunity cost of depletion. For a competitive firm manufacturing a reproducible good, the conditions for a profit maximum are to choose output such that $p(=MR) = MC$. The nonrenewable resource analogue requires $p = MC +$ the opportunity cost of depletion. How then would the mine owner measure this opportunity cost? It is the value of the unextracted resource, a resource rent related to those discussed in Chapter 2.

The second feature that differentiates nonrenewable resources from reproducible goods concerns the *value of the resource rent over time.* Deciding how quickly to extract a nonrenewable resource is a type of investment problem. Suppose one has a fixed amount of money to invest in some asset, be it a savings account, an acre of land, a government bond, or the stock of a nonrenewable resource in the ground. Which asset is purchased (and held on to over time) depends on the investor's expectation of the rate of return on that asset—the increase in its value over time. The investor obviously wants to purchase the asset with the highest rate of return. However, in a perfectly competitive environment with no uncertainty, all assets must, in a market equilibrium, have the same rate of return.

To see how this is so, consider what would happen if the economy had two assets, one that increased in value 10 percent per year, the other at 20 percent per year. Assume there is no risk associated with either asset. No one would invest in the asset earning only 10 percent; everyone would want the asset earning 20 percent. The price of the high-return asset would then increase, and the price of the low-return asset would decrease until their rates of return were equalized.



What exactly is the rate of return to a nonrenewable resource? The rate of return to the mine is the resource rent—the value of the ore in the ground. When there is a *positive* discount rate, the rent is positive and rises in nominal value as depletion occurs. If the resource rent did not increase in value over time, no one would purchase the mine, because the rate of return on alternative assets would be more valuable. In addition, the owner of an existing deposit would attempt to extract all the ore as quickly as is technically feasible. Why should one hold on to ore in the ground that is increasing in value at a rate less than can be earned on, say, a savings account? Alternatively, if the value of the ore is growing at a rate in excess of what one could earn in an alternative investment, there is no incentive to extract at all. Ore left in the ground is then more valuable to the mine owner than ore extracted. To have mineral extraction then, the rental value of the mineral must be growing at the same rate as that of alternative assets.

There is one final condition imposed on the mine owner that does not occur with reproducible goods. The total amount of the natural resource extracted over time cannot exceed its total stock of reserves. We call this the *stock constraint.*

Let us draw these strands together for the first time. Suppose a mine owner has a plan of quantities extracted roughly worked out by a rule of thumb. It remains to make the extraction plan somewhat "tighter." Should he extract one more ton of ore in this year's liftings or leave it for next year's liftings. If he *takes it out this year* and gets \$10 profit, he can put that profit (rent) in the bank at, say, 8 percent and have \$10(1.08) = \$10.80 next year. If he leaves it in the ground and *takes it out next year,* he foresees that he will get a different price and can reap a profit (rent) of \$11. In this case he will make more money by deferring extraction of the extra ton until next year. (If he were to get only \$10.75 upon extracting next year, it would pay to extract currently and sell the ore this year.) By doing this calculation repeatedly the mine owner arrives at the best extraction plan year by year. Let us now examine the important features of nonrenewable resource extraction in more detail.

### Extraction from a Mine Facing a Constant Price

One of the earliest economic analyses of mineral extraction appeared in 1914 in an article by L. C. Gray. In Gray's model, the owner of a small mine has to decide how much ore to extract and for how long a period of time. To solve this problem, Gray made a number of simplifying assumptions. First, he assumed that the market price of a unit of the mineral remained constant (in real terms) over the life of the mine. The producer knew the exact amount of reserves in the mine (the stock) prior to extraction. All the ore was of uniform quality. *Extraction costs then depended only on the quantity removed.*

We could view Gray's mine as a gigantic block of pure copper. Price per ton is constant forever, while the marginal cost of cutting off a piece of copper rises with the size of the piece cut off. If 1 ton of copper is cut off, it will cost \$500 to remove. If 10 tons are cut off at once, the extraction costs could be \$10,000. The economic problem is to cut off appropriate quantities in each period in order to maximize the present value of profits available from the stock of the mineral. The model has practical appeal, because in many mineral markets we do observe relatively constant prices over long periods of time.

To determine the efficient extraction path for the mine, we start with a simple illustration. Suppose the mine will operate for two periods only. The mine owner must determine how much copper to chop off the block today and tomorrow, using the three conditions identified earlier.[2] For the two-period case, these conditions can be stated as:

1. Price = $MC$ + rent in each period (in present value).
2. Rent today = the present value of rent tomorrow.
3. Extraction today + extraction tomorrow = total stock of reserves.

The solution is shown in Figure 3.1. Assume that the mine has a U-shaped average cost curve $(AC)$ and an upward-sloping marginal cost curve $(MC)$ over some output range.[3] The constant price is shown as $p$. Output today is designated $q(0)$, output tomorrow is $q(T)$, where $T$ signifies the end of the mining operation—the length of time the mine operates (in this case, two periods). Given these curves and the total stock of ore, there will be a unique solution to the extraction problem that satisfies all three conditions.

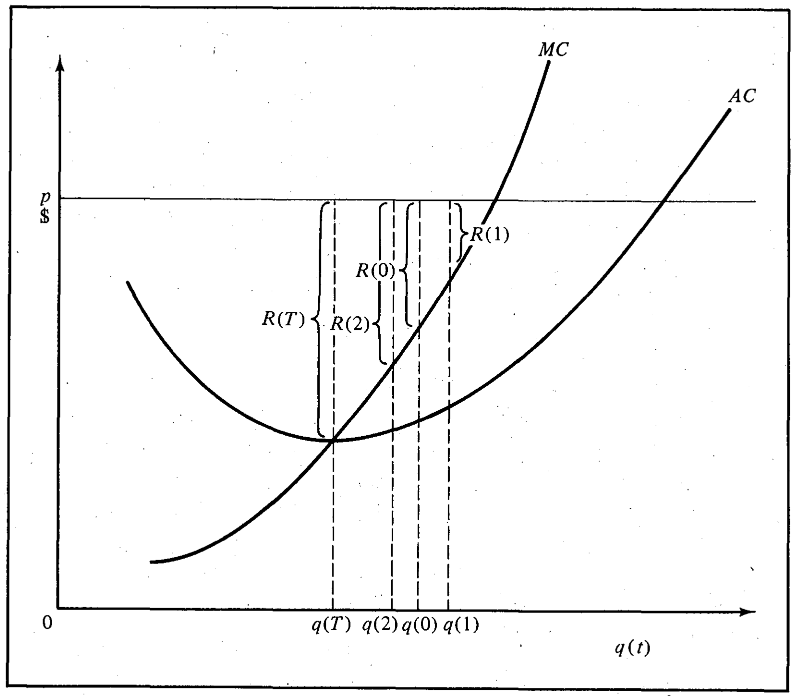



**Figure 3.1** Mineral extraction in two periods when there is a constant price. The mine operator extracts the amount $q(0)$ today and $q(T)$ tomorrow. The sum of $q(0)$ plus $q(T)$ completely exhausts the mine's reserves. Rents are $R(0)$ today and $R(T)$ tomorrow, where it must be the case that $[R(0)](1 + r) = R(T)$, where $r$ is the interest rate on alternative assets. Output levels $q(1)$ and $q(2)$ illustrate a plan that is not feasible because $q(1) + q(2)$ exceeds the total stock of reserves.



The mine owner must pick an initial output level where $p = MC$ + rent. The resource rent obtained at the output level $q(0)$ is $R(0)$. This is condition 1. Notice that condition 1 defines rent as the difference between price and marginal cost. In the next period, extraction must equal $q(T)$ and the rent will be $R(T)$. It must be the case that $R(0) = R(T)/(1 + r)$, where $r$ is the "market" interest rate or discount rate, the rate of return on any alternative asset.[4] This is condition 2. If rents did not rise at the rate of interest, extraction would not occur in both periods. If rent rose more slowly than the interest rate, the entire stock of ore would be extracted in the initial period and the proceeds of the sale invested in some other assets whose value would rise at the rate of interest (e.g., a savings account). If rent rose faster than the rate of interest, the entire stock of ore would be held in the ground until the last moment in time and then extracted. In this case, the mine is worth more unextracted because the rate of return on holding ore in the ground exceeds the return on alternative investments. Unless the rental value of the mine is growing at *exactly the same rate* as the value of other assets, extraction will either be as fast as possible or deferred as long as possible. Finally, output today and tomorrow must be chosen such that $q(0) + q(T) = S$, where $S$ is the stock of mineral reserves. This is condition 3. For a given $S$, $r$, and $p$, there will be only one level of initial output and hence final output that satisfies all these conditions.

To see that $q(0)$ is unique, consider a case where the mine owner selects an initial output level greater than $q(0)$, say $q(1)$. The rent will then be $R(1)$ which is less than $R(0)$. Output in the second period must then be such that $R(1) = R(2)/(1 + r)$. This occurs at output $q(2)$. The mine owner will then have satisfied two of the three conditions (1 and 2), but notice that condition 3 is violated. The sum of $q(1) + q(2)$ must exceed $S$ because they are both larger than previous outputs chosen. This extraction plan simply is not possible. The owner cannot extract more ore than exists in the mine. Suppose the sum of $q(0) + q(T)$ is less than $S$. Then the manager has ore remaining in the ground after extraction ceases, and revenue will be lost on the unextracted ore. A slightly higher extraction rate would yield additional profits.

This example can easily be extended to many periods of operation, but the same three conditions must be met. In addition, we can also tell when the mine will cease operation—how long $T$ is. Refer again to Figure 3.1. It is not a coincidence that $q(T)$ is at the point where $MC = AC$. This point is called a *terminal condition* for the nonrenewable resource extraction problem. It has a clear economic interpretation. Consider any output level to the right or left of this point. If output in the final period is to the right of $q(T)$, the last unit of the mineral extracted will yield a marginal rent of $p - C'[q(T)]$ where $C'[q(T)]$ is marginal cost at $q(T)$. Each ton of the mineral mined in the last period will contribute an average rent of

$$\frac{pq(T) - C(q(T))}{q(T)}$$

or $p - AC$. By inspection, we can see that the average rent exceeds the marginal rent. It would therefore increase the present value of profits (rents) if the mine manager moved more tons of ore from the last period into the first period. Similarly, if marginal rent is greater than average rent at the last period $(MC < AC)$, rents would be increased by moving ore in to the last period and out of the first period. Therefore, the optimal extraction plan must have the number of tons in the last period such that average rent is equal to marginal rent; that is,

$$\frac{pq(T) - C(q(T))}{q(T)} = p - C'(q(T))$$

In terms of Figure 3.1, average and marginal costs are equal at $q(T)$, and $q(T)$ is also that output which combined with $q(0)$ exhausts the mine. In the many-period case, the time to depletion will be such that all three extraction conditions are satisfied, plus the terminal condition. This will determine a unique $T$. Table 3.1 provides a numerical example of the efficient extraction path in a many-period situation. Price is constant at \$10, and marginal cost rises at a 45-degree line from $MC = 1$ when quantity extracted in zero. In this example, 14.8 tons are extracted over seven periods, with quantity declining toward the final period of extraction: $q(0) = 3.9197$, $q(1) = 3.4117$, $q(2) = 2.853$, . . . . Between consecutive periods $(p - mc)(1 + r) = p - mc$ where $r = 0.1$, a 10 percent rate of interest. This is our condition that rent rises at the rate of interest. In the last period $p - mc = p - ac$, which is the condition for extracting the optimal amount in the last period (the terminal condition). Total profits evaluated in present value in period 0 are \$94.4604.

## Profit Maximization for the Mine[5]

*Profit maximization* involves making revenues large in relation to costs of production. There is a series of revenues minus costs each year or period into the future. Each instant in time is slightly different, since depletion of the stock is occurring year by year. Discounting with the current interest rate makes each annual profit value comparable to others at the date at the beginning of extraction. In the absence of discounting, recall that profit in year 8 in the future would be not comparable with profit in year 11. Each nominal value is different at any one point in time in the absence of discounting.

The conditions discussed for the mine facing a constant price will also hold in this more general model. For the mine owner, total discounted profits (the present value of profits) are

$$\begin{aligned}\pi = p \cdot q(0) - C(q(0)) + [p \cdot q(1) - C(q(1))]\left(\frac{1}{1+r}\right) \\ + [p \cdot Q(2) - C(q(2))]\left(\frac{1}{1+r}\right)^2 \\ + \cdots + [p \cdot q(T) - C(q(T))]\left(\frac{1}{1+r}\right)^T \qquad (3.1)\end{aligned}$$

Equation 3.1 is what the mine owner wants to maximize subject to the stock constraint which requires that

$$q(0) + q(1) + \cdots + q(T) \leq S \qquad (3.2)$$

**Table 3.1 EXTRACTION FROM A MINE***

| $t$ | $p - mc$ | $mc$ | $q$ | $pq$ | $ac$ | $(p - ac)q$ | $\frac{(p - ac)q}{(1.1)^t}$ |
|---|---|---|---|---|---|---|---|
| 6 | 9 | 1 | 0 | 0 | 1 | 0 | 0 |
| 5 | 8.181818 | 1.818182 | .818182 | 8.18182 | 1.409091 | 7.028927107 | 4.364410719 |
| 4 | 7.4380165 | 2.5619835 | 1.5619835 | 15.619835 | 1.7809918 | 12.83795519 | 8.768496134 |
| 3 | 6.761833 | 3.238167 | 2.238167 | 22.38167 | 2.1190835 | 17.63880724 | 13.25229695 |
| 2 | 6.1471211 | 3.8528789 | 2.8528789 | 28.528789 | 2.4264395 | 21.60645095 | 17.85657103 |
| 1 | 5.5882919 | 4.4117081 | 3.4117081 | 34.117081 | 2.7058541 | 24.88549665 | 22.62317877 |
| 0 | 5.0802654 | 4.9197346 | 3.9197346 | 39.197346 | 2.9598673 | 27.59545173 | 27.59545173 |
| | | | 14.802654 | | | | 94.46040533 |

* A stock of 14.8 tons is extracted over seven periods in order to maximize the present value of profit (which ends up at \$94.46). Price is constant at \$10 and marginal extractions costs are linear and increasing. We solve by working back from period 6 to period 0. (Parameters: $r = 0.1$, $p = 10$, $mc = 1 + q$, $ac = 1 + 0.5q$, $S = 14.802654$.

Equation (3.2) says that the sum of the quantities of ore extracted must not exceed the total stock of reserves available. The symbols used in Equations (3.1) and (3.2) are defined as follows:

$p$ is the constant price per ton for the mineral
$q(t)$ is the quantity extracted in time $t$
$C[q(t)]$ is the total cost of extracting $q(t)$ tons of the mineral
$t$ indicates the time period. Today is time 0, the next period is time 1, and so on. One can think of these periods as years
$r$ is the discount or interest rate, which is assumed to remain constant over time
$T$ is the number of periods over which the mine will be operated
$S$ is the total stock of mineral reserves

All variables are interpreted in real (constant dollar) terms.

Maximizing this profit stream subject to the stock constraint on total output yields

$$\begin{aligned} p - c'(q(0)) &= k \\ \left(\frac{1}{1+r}\right)[p - c'(q(1))] &= k \\ &\vdots \\ \left(\frac{1}{1+r}\right)^T [p - c'(q(T))] &= k \end{aligned} \tag{3.3}$$

where $c'$ means $dC/dq$ and $k$ is a constant dependent on the stock size $S$ ($k$ is called the shadow price of a unit of stock). The principle in action here is that the discounted value of the marginal ton taken out in any period must be the same for an extraction program to be profit-maximizing. If this were not the case, the mine operator could increase the return to the mine by shifting production to where the marginal ton earns a higher discounted value. $p - c'(q(t))$ is the value of the marginal or last ton taken out in period $t$. $(1/1+r)^t[p - c'(q(t))]$ is its discounted value.

For adjacent periods in time we have

$$\left(\frac{1}{1+r}\right)^t [p - c'(q(t))] = \left(\frac{1}{1+r}\right)^{t+1} [p - c'(q(t+1))]$$

or

$$\frac{[p - c'(q(t+1))] - [p - c'(q(t))]}{[p - c'(q(t))]} = r \tag{3.4}$$

which says that the percentage change in $p - c'$ between periods must equal the rate of interest. $p - c'$ is the *rent* on the marginal ton extracted. So we have the basic efficiency condition: *The percentage change in rent across periods equals the rate of interest.*

The terminal condition requires that the quantities chosen are those which maximize discounted total profits so that the average profit in the last period equals the marginal profit on the last ton extracted. This tells us how to terminate the sequence $q(1)$, $q(2)$, $q(3)$. This condition combined with the stock constraint makes sure that the sum of the $q$s equals the original stock. These two conditions are

$$\frac{pq(T) - C(q(T))}{q(T)} = p - c'[q(T)] \tag{3.5}$$

and

$$q(1) + q(2) + \cdots + q(T) = S \tag{3.6}$$

These two conditions, in addition to the percentage change in rent condition, yield the profit-maximizing number of periods over which to exhaust the given stock.

Now we turn to mines with an ore quality which declines as extraction moves deeper into the mineral material. We are going to associate costs of extraction and processing with each unrefined ton, not with a "batch" of homogeneous stock as we did above. We no longer have a large homogeneous block of copper to cut away at.



### Quality Variation Within the Mine

In the previous section, it was assumed that the cost of extraction rose only if more units of ore were extracted at any one time. Suppose now that the ore is not a chunk of pure copper as before, but consists of metal and waste rock. The metal is distributed throughout the waste rock in seams of varying thickness. The mine owner would like to extract from the thickest seams first, where the ratio of metal to waste rock is the highest. Suppose the deposit is laid down with richest seams on top. As the ore body is mined, the thicker seams are depleted and more waste rock must be removed to get at increasingly thinner seams. Mining costs rise per unit of metal produced simply because the metal content of the ore diminishes while the rock content increases. This means that the marginal cost of extracting and processing *each ton* of ore is different.

Extraction costs per ton shift up (increase) as subsequent amounts of ore are extracted. The flow condition for efficient extraction of the mineral (conditions 1 and 2, or Equation 3.4) is unchanged, but now holds for a single ton of ore of a specific quality (seam thickness). The mine owner can no longer slide down the marginal cost curve by extracting smaller amounts of ore over time to satisfy the conditions for efficient extraction because the marginal cost of extraction increases (shifts up) for each incremental ton of ore processed. To see what happens to the extraction path in this case, we turn to Figure 3.2.

The mine represented in Figure 3.2 illustrates a case where ore quality is continuously decreasing, and we are examining extraction over two periods of time. There are thus two curves of extraction cost per ton, one for period $t$ and another for period $(t+1)$, where the curve for $(t+1)$ lies everywhere above that for period $t$, indicating that it will cost more to extract and process additional units of ore over time. The mine owner must determine how much ore to extract in periods $t$ and $(t+1)$ by following the flow condition. The quantity extracted in $t$ must be chosen such that the rent on the last ton in the period will be exactly equal to the rent that ton could obtain if extracted in the next period, discounted by $(1+r)$. Or, in terms of Figure 3.2, the rent on the marginal ton in the first panel of the figure is the amount *ab* if output is chosen at $q(t)$ and the price is $p(t)$. If the mine owner is to be indifferent between extracting the marginal ton in period $t$ or in period $(t+1)$, it must be the case

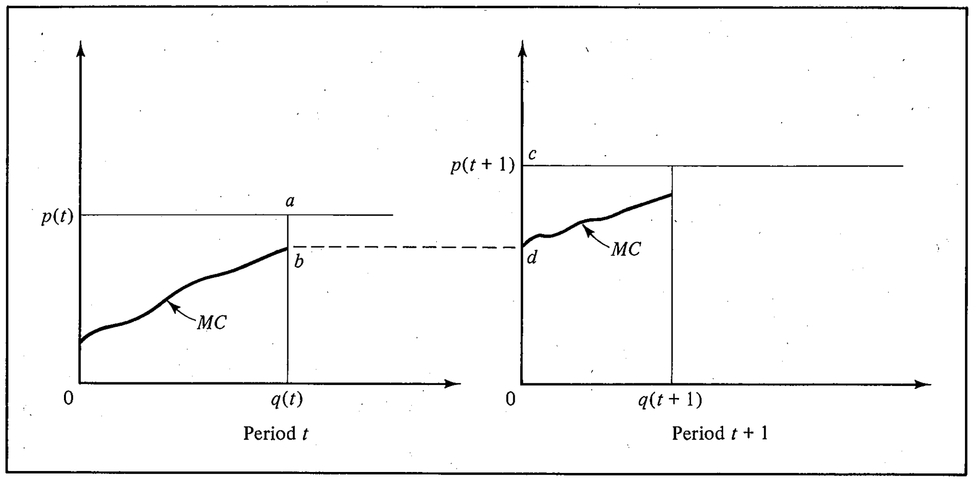



**Figure 3.2** Quantity $q(t)$ in period $t$ is set so that distance $ab$ equals distance $cd$ divided by $1 + r$. The marginal ton in period $t$ gets rent $ab$ and would get rent $cd$ if it were extracted in period $t + 1$.

that the rent in $(t + 1)$ is equal to $cd$, where $cd = ab(1 + r)$. The marginal ton in period $t$ is, we emphasize, the "least marginal" ton in period $(t + 1)$ because now every ton is of a different quality or has a different extraction and processing cost. This is what is illustrated by the higher marginal cost curve in the second panel of Figure 3.2.

The important implication of this analysis is that the market price must rise over time if extraction of lower-quality, higher-cost ore is to occur. If the mineral price does not rise to $p(t + 1)$ in the period $(t + 1)$, no extraction will occur in that period. Extraction will end with $p(t)$ equal to the extraction cost on the last ton taken out in period $t$. This possibility gives rise to another important distinction about the end of the mining operation. In the case of uniform ore quality, we argued that mining would cease when all the ore was removed. We can call this *physical* depletion or exhaustion.

Suppose, however, that the ore is not of uniform quality and the costs of extracting additional units rise, as Figure 3.2 shows. If the market price does not rise sufficiently to ensure that extraction proceeds from one period to another, the mine will shut down. If in period $(t + 1)$ the price is constant at $p(t)$, ore will be extracted to the point that $p(t)$ equals extraction cost. The mine is then said to have *economic* depletion in period $t$. It simply does not pay the mine owner to extract any ore beyond the quality indicated at $q(t)$, given the extraction cost curves and the price $p(t)$. A higher rate of return can be earned by taking the rent, $ab$, and investing it in an alternative asset which earns the market interest rate of $r$ percent per year.

In a many-period model, the length of the extraction period will be determined by the time path of prices. For two prices in any consecutive periods, there will be only one value of cost per ton and hence output that satisfies the flow condition. The optimal life of the mine, the length of time to depletion (whether economic or physical), will then be determined by linking together quantities over subsequent periods until the flow condition no longer is satisfied, or the mine runs out of ore.



Mines frequently have valuable by-products. The mining of nickel in Canada yields profitable amounts of gold and platinum. It is straightforward to amend our model of the mine to incorporate this situation. Valuable rock, $S_0$, is defined such that for each scoop of size $Q$, there is $K(Q)$ of nickel and $G(Q)$ of gold. We continue to suppose that costs rise with the amount of rock, $Q$, processed. For $p_K$ the net revenue for nickel, and $p_G$ for gold, we get a revised pricing rule for optimal extraction:

$$\left(\frac{1}{1+r}\right)^t \left(p_K \frac{dK}{dQ(t)} + p_G \frac{dG}{dQ(t)} - \frac{dC}{dQ(t)}\right) = \text{constant for all } t$$

Now a weighted sum of prices net of costs increases at the rate of interest for the mine owner to be maximizing the discounted present value of profit: $dC/dQ(t)$ is the marginal cost of hoisting quantity $Q(t)$.

We have seen that when quality variation is introduced in either of two forms above that a simple rule such as "rent rising at the rate of interest" must be amended in an essential way. In other words, simple formulas are inadequate for characterizing real world extraction programs. Finally, note that because Gray's model is for a single firm, there is no discussion of the resource market and how (or if) mineral prices will rise to satisfy the flow condition and allow production to occur over time. To examine this important issue, we now turn to a model of a mineral industry.

## EXTRACTION BY A MINERAL INDUSTRY

### The Hotelling Model

In 1931, Harold Hotelling wrote a classic paper which examined the optimal extraction of a nonrenewable resource from the viewpoint of a social planning agency that had as its goal the maximization of social welfare from the production of minerals. The model was at the industry level rather than that of the single mine. Both Gray and Hotelling arrived at the same condition for the efficient extraction of a mineral—namely, that the present value of a unit of a homogeneous but finite stock of the mineral must be identical regardless of when it is extracted. This principle reflects conditions 1 and 2, which we call the *flow condition*. Together with the stock constraint and terminal condition, the optimal extraction plan for the nonrenewable resource can be determined at the industry level as well as for the single mine.

Hotelling viewed the problem of how to extract a fixed stock of a natural resource from the vantage point of a government social planning agency. He then showed that a competitive industry facing the same extraction costs and demand curve as the government, and having perfect information about resource prices, will arrive at exactly the same extraction path for the mineral.[6] The efficient extraction path determined by each firm acting independently in the competitive industry will yield the socially optimal extraction path. We first examine the planner's solution, and then show why it is achieved in a competitive industry. As before, a number of simplifying assumptions are made and then gradually modified to illustrate cases with practical relevance.

When we deal with an industry rather than a single mine, the mineral price can no longer be treated as a constant. Rather, it is assumed that the industry faces a negatively-sloped demand curve. The greater the industry output, the lower the price will have to be if we are to have an equilibrium in any mineral market at any given point in time. Hotelling assumed that prices would adjust so that a mineral market would be in equilibrium at every point in time; supply must always equal demand.[7] We can think of the Hotelling model as examining world production of oil, nickel, copper, or some other mineral. As before, the stock of mineral reserves is of known size, and all units of the mineral are homogeneous. We assume a unit of the stock costs $c$ dollars to extract and refine and that this cost is constant for all units of the stock in the reserve endowment, $S$. Once again, it is as if we had a huge block of copper that cost \$$c$ per ton to chip off. We want to find the rate of extraction that maximizes social welfare and completely exhausts the stock. (If $c = 0$, the analysis is qualitatively unchanged. If $c$ increases with the quantity extracted, we are back with Gray's cost assumption, which is an unnecessary minor complication at this point. See question 4 at the end of the chapter.)

The problem is to maximize society's wealth ($W$) or net return from mineral extraction, which is defined as

$$W = B(q(0)) + B(q(1))\left(\frac{1}{1+r}\right) + B(q(2))\left(\frac{1}{1+r}\right)^2 + \cdots + B(q(T))\left(\frac{1}{1+r}\right)^T \tag{3.7}$$

Again, there is the resource stock constraint which requires that

$$q(0) + q(1) + q(2) + \cdots + q(T) \leq S \tag{3.8}$$

$B(q(t))$ is the consumer plus producer surplus obtained in period $t$ from the extraction of output, $q(t)$. This social surplus is simply the area under the demand curve up to quantity $q(t)$ and above the constant costs of extracting $q(t)$.

This problem can be solved in a manner analogous to that presented previously. As before, the solution requires that the flow condition, terminal condition, and stock constraint be met. The crucial distinction between Hotelling's model and Gray's is that we must examine the demand curve explicitly and derive a unique price for the resource in each period the mineral industry operates. In addition, we can make the stronger statement about the socially optimal as opposed to just the mine's efficient extraction path. We now derive the solution to the maximization of Equation (3.7) subject to the stock constraint, Equation (3.8), for a linear demand curve.

In maximizing social welfare, the planner must decide what the net benefits are of extracting some of the mineral today as opposed to tomorrow. Therefore, the planner will want to measure the change in the social surplus as one more unit of the mineral is produced today. Consider Figure 3.3. The social surplus for the last unit extracted is simply the difference between the market price and the marginal cost of extraction, $c$. If $q(t)$ is extracted in period $t$, society gains the amount for the $q(t)$th unit extracted and the amount under the demand curve and above $c$ for all previous or inframarginal units extracted. How much will the planner choose to extract in each period?

As before, the flow condition provides an answer. To maximize social welfare, it must be the case that the net benefit to society from the last unit extracted in each



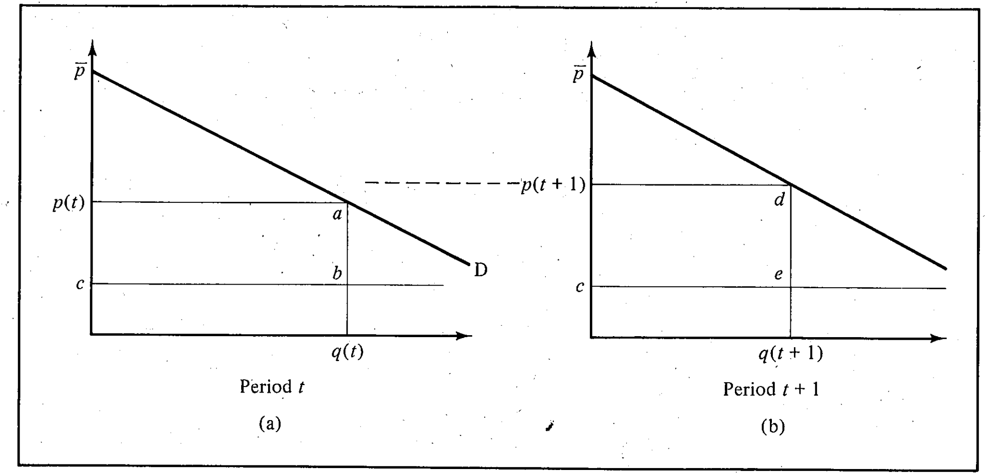



**Figure 3.3** Rent per ton in period $t$ is distance $ab$ and rent per ton in period $t+1$ is distance $de$ where $ab(1+r) = de$. Price is higher in period $t+1$, making $[p(t) - c](1+r) = [p(t+1) - c]$ and this rent path yields the optimal path of $q$'s to extract.

period is exactly equal in present value terms in each period of extraction. To do otherwise would entail foregoing the maximum benefits possible. And because the net benefit of the marginal unit extracted is simply the resource rent ($ab$), it must be the case that the present value of the rent on the margin in each period must be equal. For the two-period case, $q(t)$ and $q(t+1)$ must be chosen such that

$$p(t) - c = [p(t+1) - c]\left(\frac{1}{1+r}\right) \tag{3.9}$$

Figure 3.3 illustrates one pair of outputs for the two periods which will satisfy this condition given $D$, the demand curve, and $c$ and $r$. Notice the flow condition implies, as can be seen in Figure 3.3, that the mineral price must rise over time. In this model, with a stationary demand curve, the only way the price, and hence the rent, will rise is if the quantity extracted declines over time. Therefore, extraction in period $(t+1)$ must be less than that in period $t$ to ensure that the price rises just enough to satisfy Equation (3.9).

We can rewrite Equation (3.9) to obtain

$$\frac{[p(t+1) - c] + [p(t) - c]}{[p(t) - c]} = r \tag{3.10}$$

Written in this form, we see that as price rises, *rent per ton grows over time at a rate equal to the rate of interest.* This is often referred to as Hotelling's $r$ percent rule, or simply *Hotelling's rule.* We sketch the price path in Figure 3.4. How do we know the path shown is the one that maximizes social welfare? All Hotelling's rule says is that rents must grow at the rate of interest. Might there be dozens of different paths, all of which satisfy Hotelling's rule? Yes, but a unique path of output can be derived with the help of the stock constraint and terminal condition. If extraction costs are constant, the planner will want to ensure that all the mineral is removed. If any ore is left

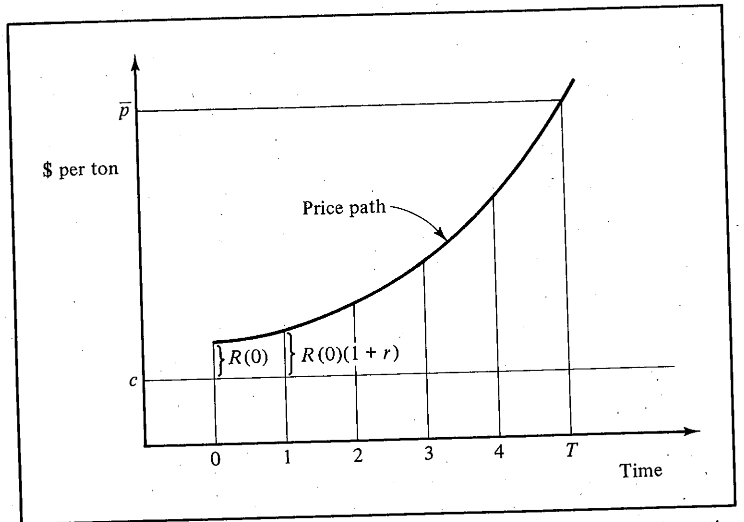



**Figure 3.4** Price minus cost $c$ is rising between periods at a rate equal to $r$, the rate of interest. This yields a rent path rising exponentially at rate $r$ in the optimal program of extraction.

in the ground, the mine owner will be foregoing rents. The constant cost assumption is crucial in this argument. Each unit of ore costs the same to extract (in nominal terms); therefore, it cannot pay the planner to leave ore behind. We know that the sum of the amounts extracted in each time period must exactly equal the total stock of the mineral reserves ($S$).

We can also see, from Figure 3.3, that with the linear demand curve there is some price, call it $\bar{p}$, at which no one is willing to buy more of the mineral. The price $\bar{p}$ is often called the *choke price*, meaning that demand for the good is choked off at this point. Ideally, the planner would seek to have the stock of the mineral go to zero at exactly the point that demand goes to zero. Therefore, the planner would seek to have the last unit of output extracted at $\bar{p}$. To do otherwise deprives society of maximum benefits.

We can then work backward from $\bar{p}$, given the fixed stock $S$, to find just that initial output $q(0)$, which will, over time, decline so that rent increases at rate $r$ and outputs sum to the stock of reserves. Only one such extraction and hence rent path exists. It will yield the largest amount of social surplus available to society and hence be the optimal plan. In addition, we can now determine the length of time the mine operates. In Figure 3.4, the point at which the price path intersects price $\bar{p}$ will determine the unique duration of the extraction profile, $T$. Once the price reaches $\bar{p}$, there will be no more demand for the mineral, so extraction will cease. Extraction ends at time $T$.

Note also that the terminal condition discussed earlier is also met at $T$. *The rent on the marginal ton must be equal to the rent on the average ton at T.* This condition also implies in this case that output at time $T$ must equal zero. Table 3.2 gives a numerical example of an optimal extraction path for the industry. Rent per ton is $p(t) - c$ and for consecutive periods $(p(t) - c)(1 + r) = p(t + 1) - c$. This is the condition that rent rises at the rate of interest. The demand curve intersects the vertical



**Table 3.2 AN INDUSTRY EXTRACTION EXAMPLE***

| $t$ | $p(t) - c$ | $p(t)$ | $q(t)$ | $(p - c)xq$ | $\frac{(p(t) - c) \cdot q(t)}{(1.1)^t}$ |
|---|---|---|---|---|---|
| 6 | 9 | 10 | 0 | 0 | 0 |
| 5 | 8.181818 | 9.181818 | .818182 | 6.6942162 | 4.1565816 |
| 4 | 7.4380165 | 8.4380165 | 1.5619835 | 11.618059 | 7.9352906 |
| 3 | 6.761833 | 7.761833 | 2.2388167 | 15.134111 | 11.370482 |
| 2 | 6.1471211 | 7.1471211 | 2.8528789 | 17.536992 | 14.493382 |
| 1 | 5.5882919 | 6.5882919 | 3.4117081 | 19.065621 | 17.332382 |
| 0 | 5.0802654 | 6.0802654 | 3.9197346 | 19.913292 | 19.913292 |
| | | | 14.802654 | | 75.20141 |

* A stock of 14.8 tons is extracted over seven periods resulting in the maximization of the present value of net consumer surplus. Extraction cost per ton is constant at $1 and the industry demand curve is linear. We solve by working back from period 6 to period 0. (Parameters: $r = 1$, $c = 1$, $q = 10 - p$, $S = 14.802654$.)

axis at $p = 10$, when $q = 0$. This value $p = 10$ is the *choke price*. Over seven periods, 14.803 tons are extracted. Total *profit* evaluated at period 0 in present value terms is $75.20. (We have not illustrated that in period 6; $p(6) - c = [B(q(6)) - cq\ (6)]q(6)$, or marginal welfare from extracting $q(6)$ equals average welfare. This is slightly tricky, since $q(6) = 0$ in order to satisfy this basic end point condition.)

### Exhaustibility and Welfare: Demand Curves and Backstop Technology

What would happen if the world were to run out of oil or any nonrenewable resource one day? What does the Hotelling model tell us about this occurrence? The impact of complete exhaustion on society depends on the technology of producing and using resources in production and can be reflected in the demand curve for the resource. A crucial question is whether substitutes for the resource exist or whether the resource is so necessary to the production process of other goods that once it is depleted the other goods will also cease to be produced. Our model with the linear demand curve and choke price says that a substitute exists. The choke price is that price at which the users of the good will switch entirely to the use of the substitute good. This substitute may be another nonrenewable resource such as oil shale as a substitute for conventional crude oil, or it may be a reproducible good such as solar energy.[8] If the substitute exists, society and economic systems will not collapse when the oil runs out; they will shift to the substitute commodity.

What if there is no substitute for the depletable resource? In this situation, as the available quantities of the resource dwindle, prices would begin to rise very quickly. We can characterize this with a nonlinear demand curve, say an isoelastic curve, that does not have a positive intercept[9] (see Figure 3.5). We will not have to worry about running out of the resource in this case, because we never will in finite time. From society's viewpoint, however, this is not a very desirable situation because what it suggests is that as the resource quantity extracted gets smaller and smaller, its price will rise to higher levels. We can think of extracting oil by the bucket, then the cup, and finally by teaspoons and eyedroppers while the price climbs continuously toward infinity. This is asymptotic depletion. (*Asymptotic* means that two lines approach each other more and more closely as time passes but never touch.) This can happen in

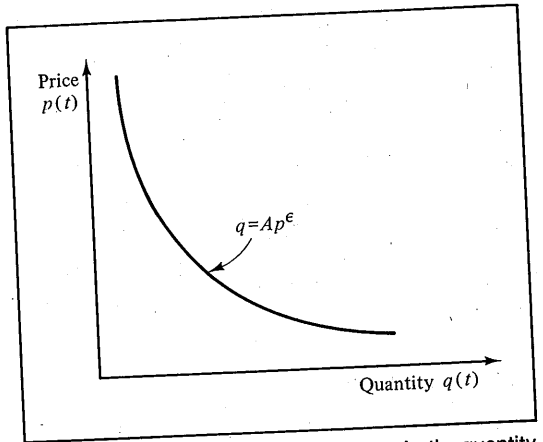



**Figure 3.5** An isoelastic demand curve. As the quantity extracted diminishes, the price rises toward infinity. Exhaustion does not occur in finite time.

a mathematical model, but it is rather unrealistic. The conventional view is that exhaustion of most minerals is followed by the production of a substitute product.

We can incorporate the substitute product into the extraction model. It is common to think of the substitute product as a *backstop technology,* a technique for producing energy at a constant cost such as fusion power that becomes feasible to implement once the price for conventional oil reaches a certain level. *Feasible* here means that the producers of the backstop can cover their costs.[10] Suppose the backstop technology can provide the substitute commodity at a price of $Z per ton forever, because the substitute can be produced with constant costs. This case is illustrated in Figure 3.6.

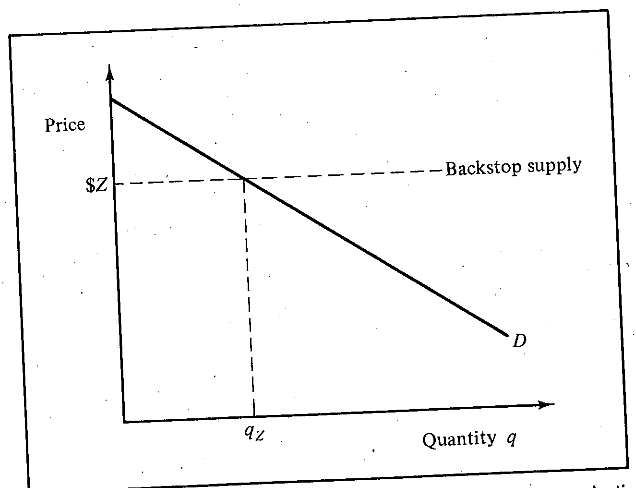



**Figure 3.6** The backstop technology provides a resource substitute at $Z per ton. Rent will rise to the point at which price per ton equals $Z and the substitute will take over as exhaustion occurs at that "joining" of price and $Z.



All demands below $Z are satisfied by flows from the nonrenewable resource stock and those at $Z are supplied by the backstop technology. One can think of the demand curve in Figure 3.6 as one for energy, derived from conventional oil wells. The backstop is energy from fusion or solar power (two possible substitute goods). For a planner controlling both sources of supply—the exhaustible stock and the backstop technology—the optimal program will be to consider $Z as the choke price for conventional oil and to arrange to exhaust this oil in the Hotelling fashion, as set out in the previous section. At the moment of exhaustion, price will have risen to $Z per ton and the backstop technology brought on line.[11]

### A Model of a Competitive Nonrenewable Resource Industry

We have argued that the socially optimal extraction path would be obtained if a planner organized production in the industry. Would a decentralized, competitive industry replicate the socially optimal program of extraction? Suppose there were a large number of mines or oil wells, each owned by a different person. If none of the owners coordinated their actions, we would have what is called a decentralized competitive setting. Each owner would be faced with this decision: Should I mine and sell a ton of ore this period and earn $p(t) - c$ dollars of profit, or should I wait until the next period and extract, sell, and receive $p(t+1) - c$ dollars of profit for the ton? If the prices $p(t)$ and $p(t+1)$ were such that Equation (3.9) was satisfied—that is, $p(t) - c = (p(t+1) - c)/(1+r)$, each owner would be indifferent between selling this period or next. Since all owners and deposits are identical (there are no quality differences among mines), all are indifferent.

If there was a general tendency to wait until next period by sellers, current output would fall and the current price would rise. Sellers would then find it profitable to sell now and put their rents (or profits) into an asset earning $r$ percent. If there was a tendency to sell a lot of ore in the period, however, the current price would fall, and mine operators would be reluctant to sell until future periods. Therefore, the flow condition will be met by each firm seeking to maximize profits to ensure extraction in each period, and the market forces of supply and demand will ensure that the condition is met.

Will a competitive industry which is on an equilibrium path also satisfy the optimal terminal condition? To show that it does, we assume that the extraction and price paths were not optimal then argue that this cannot happen in the context of our model. If the price path were not the optimal path, there would be a jump in the rent up or down at the transition to the backstop technology. Consider Figure 3.7. Extraction commences at $t = 0$ with an initial price of $p(0)$. The rent in the first period, $R(0) = p(0) - c$, grows at rate $r$ until the resource is exhausted at time $T$. But notice that at $T$, the market price of the resource appears to have risen above $\bar{p}$. This cannot occur, because with the backstop technology, no consumer will pay more than $\bar{p}$ for the resource. It therefore means that the resource will be economically depleted at time $T'$ which is less than $T$.

But this cannot be a plan that maximizes profits, because ore will be left in the ground that could have been extracted. If all firms know that this price path is emerging, they will alter their extraction plans to shift production to the present. The

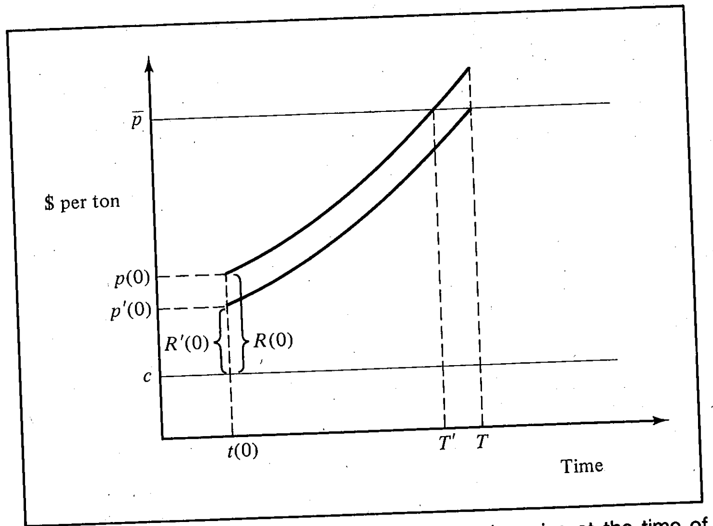



**Figure 3.7** A price path with a jump down in the price at the time of exhaustion cannot occur. Extraction would be shifted away from later periods and into earlier periods to prevent economic depletion at $T'$.

increase in current production will lower the initial price to say $p'(0)$, which yields the rent $R'(0)$. It will grow at rate $r$ and have exhaustion occur at precisely $\bar{p}$. A similar argument applies if the price path fails to reach $\bar{p}$, that is, firms run out of ore before the backstop technology can be implemented.

What these arguments require is that all mine operators have *perfect foresight* about the price of the mineral in all periods. To be able to see that it is profitable to shift production from one period to the other, individuals must know what the price will be. Given that mines can produce for several decades, it is difficult to imagine that perfect foresight characterizes the centralized or decentralized operation of mineral markets. Some economists have argued that market forces will ensure that a perfect foresight equilibrium will be achieved in decentralized situations, even if some participants in the market have deficient information. If person $i$'s foresight is

**BOX 3.1**
**Exhaustible Resource Prices**

Heal and Barrow examined a hundred-year sample of exhaustible resource prices and found that interest rates did have a significant effect on price movements but not as in the Hotelling model. They found *changes* in interest rates were significant rather than levels and indicated their results supported a "rather more general asset market approach." They also found that the level of industrial output did not have a major influence on resource prices.

*Source:* G. Heal and M. Barrow, "Empirical Investigation of the Long-Term Movement of Resource Prices: A Preliminary Report," *Economics Letters*, 7 (1981), pp. 95–103.



deficient relative to person $j$'s, $j$ could make a profit by signing a contract to buy or sell from $i$ in the future at specific conditions. The informational deficiencies will be competed away by the forces of profit maximization and free entry into the industry. We now consider some modifications of the basic model, but still assume that all participants act with perfect foresight. (Imperfect information is considered in Chapter 5.)

## EXTENSIONS OF THE INDUSTRY MODEL

### Changes in Extraction Paths Under Altered Conditions

We will show how the price and extraction paths of a competitive industry are affected by: (1) a rise in the constant costs of extraction; (2) an increase in the interest (discount) rate; and (3) the introduction of taxes. In each case, we compare equilibrium paths under two different assumptions. Diagrammatic techniques will be used to derive the results.

### An Increase in Extraction Costs

In Figure 3.8, we examine the case where the costs of extraction, while still constant, are higher than initially assumed. How will these higher costs affect the extraction and price paths? In this and all subsequent cases, we look at the effects as if no extraction had yet occurred. We could modify the results if the cost (or other parameter) change occurred at some time after the mine had begun to operate. In these situations, it will matter whether the mine owner anticipated the changes or not (see

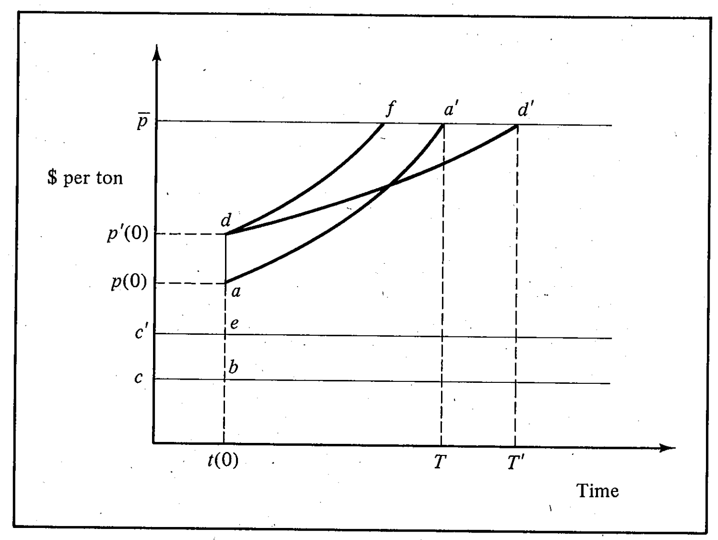



**Figure 3.8** The comparative statics of an increase in extraction costs—the effect on the price path. If costs of extraction increase from $c$ to $c'$, the mining industry will respond by reducing output in the initial period so that the resource price rises from $p(0)$ to $p'(0)$. Production is then increased again in later periods. A higher extraction cost will lengthen the time to depletion.

discussion question 7). Suppose the initial situation is an industry facing constant costs of $c$ and a choke price of $\bar{p}$ (see Figure 3.8). The price path that satisfies the flow and terminal conditions along with the stock constraint is $aa'$. Now, what would happen if costs were $c'$ rather than $c$, where $c' > c$? If the industry tried to follow the path $aa'$, it would not be maximizing profits. Along the path $aa'$, rent will grow faster than rate $r$ when costs equal $c'$. Each mine owner then decreases current output (because rents in the ground exceed returns from extraction), the industry supply declines, and the initial price, $p_0$, rises to $p_0'$ in Figure 3.8.

What happens to extraction over time? If mine owners produced less output in every period when costs are $c'$ rather than $c$, the price path would look like $df$. In this situation, the choke price $\bar{p}$ is reached when there is still ore remaining in the ground. This cannot be an optimal plan because the terminal condition is not met. What it means is that at some point in the extraction path, mines must begin to increase their rate of production, which will cause the market price to rise less rapidly than when costs were $c$. This will ensure that physical depletion occurs at $\bar{p}$. Thus, the path $dd'$ results. It is important to notice that the increase in costs results in a *lengthening* of the time to depletion. The backstop is reached at $T'$, which exceeds $T$.

There is an economically intuitive explanation for the effect on the extraction and hence the price path when costs change. If the cost of extraction is higher, in present value terms it will benefit each firm to postpone extraction. If firms postpone incurring the costs, the rents will be larger than if the same path as $aa'$ is followed. Production is reduced in the early periods and increased in later periods. The present value of the mine has fallen due to the increase in costs, but the stock of ore is still physically depleted and the terminal condition is met.

## A Rise in Interest Rates

Suppose that the rate of return on investing in assets alternative to mineral extraction rises. What will be the response of the mining firms? In Figure 3.9, suppose the price path prior to the increase in the interest rate is $aa'$. If the mine operators continued to follow this path, the mines would be earning a lower rate of return over time than available elsewhere. The way to avoid this loss is to shift production to the present. The mine owners will extract more ore in the initial period, thus driving down the market price to, say, $p'(0)$.

Thereafter, less ore will be extracted so that the rate of return on the remaining ore rises at the now higher interest rate. This means, however, that the time to depletion must fall. The extraction path $dd'$ will start from a lower initial price than did $aa'$ and will rise more steeply so that it hits the choke price at $T'$, which is less than $T$. Any other path would not maximize the profits of the mines, given the new interest rate.

## The Introduction of Taxes

Suppose the government decided to impose various taxes on the mining industry. What would be their effect on the extraction and price paths, and the time to depletion? We consider two types of taxes: a tax on the mineral rent, the difference between



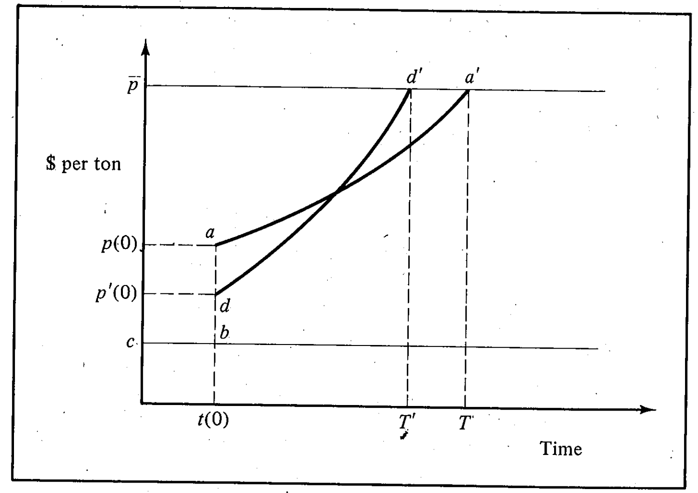



**Figure 3.9** The comparative statics of an increase in the interest rate. A rise in the interest rate will increase the opportunity cost of extracting ore in the future (the present value of future rents is lower). Extraction is shifted to the present, and the initial price falls. The smaller amounts of ore remaining in the ground increase in value at a higher rate than before, so the path $dd'$ is steeper than $aa'$. Depletion occurs at $T'$ which is sooner than $T$, the exhaustion point under a lower interest rate.

price and marginal cost, and a royalty, a tax at a constant rate, on the value of production.[12] A rent tax will have no effect on the extraction decision of a mine already in existence. There is no change in the rate of extraction over time that can offset the decline in the present value of the mine resulting from the tax. The government will simply collect some of the mineral rent, and production will proceed in the same manner as before the tax.

To see that this is so, we present the flow condition required after a rent tax is imposed. Let $\alpha$ be the tax rate levied on mineral rents or profits. Then Equation (3.9) becomes

$$(p(t) - c)(1 - \alpha) = (1 - \alpha)(p(t) - c)\left(\frac{1}{1 + r}\right) \qquad (3.11)$$

Because rent in each period is taxed exactly the same, the term $(1 - \alpha)$ cancels from both sides of Equation (3.11). There is no way the mine operator can avoid the tax by shifting production.

While a rent tax is said to be *neutral* or nondistorting to the extraction path because it does not alter the path, the same cannot be said about the effect of this tax on the discovery of new mines. The higher the tax rate, the less incentive there is for individuals and firms to explore for new mineral deposits. Two identical deposits with different taxes levied on their output represent two assets of quite different market value. The rent tax reduces the return from exploration—the value of the mineral in the ground—because the expected payoff from discovering a new deposit declines with the tax. We will come back to this topic in our discussion of exploration and uncertainty in Chapter 5.

Now let us examine the effect of the imposition of a royalty on the total value of mineral extraction. The government now taxes the total revenues of each mining firm at, say, rate $\gamma$. The introduction of the royalty has an effect analogous to a rise in the cost of extraction. If the firm postpones the extraction of some ore to the future, it can then reduce the effect of the royalty on the present value of its rents. To see this, we rewrite Equation (3.11) to obtain the flow condition after the royalty.

$$(1-\gamma)(p(t)) - c = [(1-\gamma)(p(t)) - c]\,\frac{1}{(1+r)} \tag{3.12}$$

Notice that there is no way we can cancel the term $(1-\gamma)$ from both sides of Equation (3.12). The royalty reduces the price received by the firm for each unit of mineral sold and thus, the present value of the mine. If sales are postponed, the effect of this reduction will be minimized because of the discount factor. The result is a time path of extraction similar to that shown in Figure 3.8; the initial output will fall, and price will rise. Later in time, extraction will rise again, and price will rise less quickly than in the case without a mineral royalty. Again, the time to depletion of the fixed stock is lengthened. The size of this effect depends on the magnitude of costs. If extraction cost is very small, the distorting effects of the royalty are relatively small.

Numerous other comparative static exercises can be done (see discussion problem 8). These exercises will help in understanding the model of the industry and also enable the reader to try applications of the model to some real-world events, such as the introduction of new energy taxes, a fall in the cost of producing oil from oil shale deposits, and so on. We now turn to some further extensions of the industry model that make it more compatible with real-world observations.

### Declining Quality of the Stock

Suppose the mineral industry finds its stock of ore declining in quality. Deposits of poorer quality must be brought on stream as the high-quality reserves are exhausted. As in the case of the single mine, quality decline can be viewed as requiring the removal of more waste rock per unit of ore extracted to get at the thinner seams of metal. Thus the average cost of producing metal increases as more of the mineral is extracted. If each ton has a specific extraction and processing cost associated with it, and these costs rise as more mineral is extracted, the flow condition of Equation (3.9) remains the same, but its interpretation is modified.

For a *specific ton* of the mineral, the flow condition requires the rent on that ton in period $t$ to be equal to the discounted rent on that same ton if it were extracted in period $(t+1)$. In period $t$, this ton will be the marginal ton extracted, whereas in period $(t+1)$ it will be the most inframarginal ton extracted. We illustrate the effects on the industry in Figure 3.10.

In Figure 3.10, the rent on the marginal ton in period $t$ is $ab$. This same ton would obtain the rent of $ge$ in period $t+1$. Distance $ge$ must be equal to the amount $ab(1+r)$, if the owner of the marginal ton is to be indifferent between extracting in period $t$ or period $(t+1)$. All owners of ore inframarginal to the marginal ton earning $ab$ at $t$ will extract their ore at that point simply because their rent in period $t$ exceeds



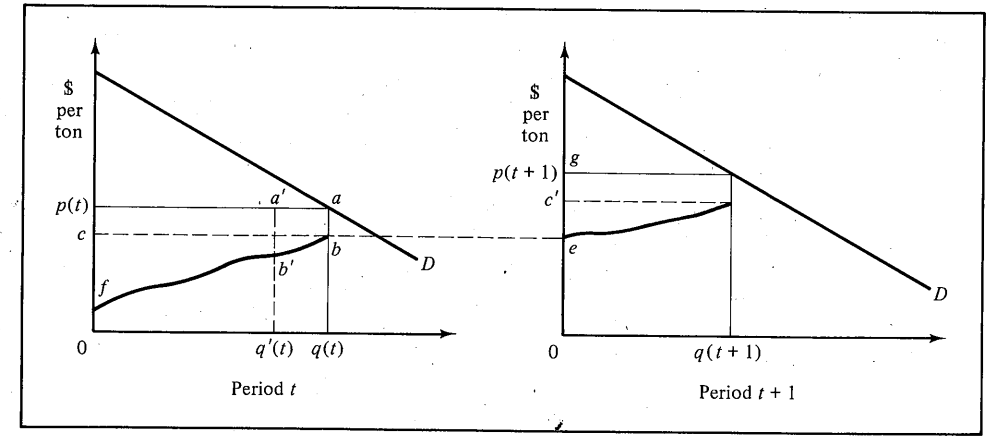



**Figure 3.10** The marginal ton extracted in period $t$ has rent per ton of $ab$ or $p(t) - c$, and this ton would earn rent $p(t+1) - c$ in period $t+1$ or distance $ge$. The rate condition "causing" $q(t)$ to be the optimal level to extract is $[p(t) - c](1+r) = [p(t+1) - c]$. $fb$ is the rising extraction cost per ton for tons of declining quality.

the rent they could obtain by waiting until period $(t+1)$. The inframarginal ton, $q'(t)$, for example, earns a rent of $a'b'$ in period $t$. It would earn $ge$ in period $(t+1)$ which is less than $a'b'(1+r)$. The flow condition for extraction in both periods is not satisfied for this ton, and hence the ton is extracted in period $t$. A similar argument applies to all tons to the left of $q(t)$ in Figure 3.10.

Distance $ab$ is the *rent* per ton. Area $cbf$ is the *Ricardian* or *differential rent* arising from the variation in ore quality. The marginal ton of ore in period $(t+1)$ earns a rent equal to $p(t+1) - c'$. If the flow condition is then linked together for all periods, the amount of ore to be extracted at each different quality will be determined for each period. The price in each period is determined by the aggregate amount extracted; thus, the price is endogenous.

We have yet to examine the terminal condition for the problem and set the time to depletion, and hence initial $q(t)$. Because there are now many grades of ore in the industry, we must distinguish between the possibility of economic versus physical exhaustion. Assume in either case that there is a linear demand curve and choke price $\bar{p}$. If there is physical exhaustion, the output will go to zero in the last period, just as the price hits $\bar{p}$. With $q(T) = 0$, marginal rent equals average rent on the last batch of ore removed. We can then work backward to find the unique first period output that satisfies the flow and terminal condition with complete physical exhaustion.

Physical exhaustion will occur in any industry if the marginal cost of extracting the final ton is less than (or equal to) the choke price. If, however, there are ore qualities with extraction costs in excess of the choke price, economic exhaustion occurs. In this situation, the mining sequence ends when the choke price equals the marginal cost of extraction. There will be an ore grade at which mining ceases. The final quantity extracted from all mines is that which yields $\bar{p} - c = 0$. Again, we work backward from this quantity to find the initial output level that satisfies the flow condition in each period.

### Deposits of Distinct and Differing Quality

Now we examine the case where each deposit has ore of a uniform quality, but differs in quality from other deposits. What is an optimal plan for exploitation of ore in this case? We can treat ore quality as signifying different costs of extraction and processing. These costs of extraction could be due to ore grade and seam thickness, as discussed earlier, or simply to the fact that deposits are located at different distances from a central market. Transportation costs give the deposits a distinct "quality"; the metal is still of uniform quality.

Consider an example of two deposits of different quality within a competitive industry. Each deposit is within itself of uniform quality. Deposit 1 has extraction costs of $c_1$ and an initial stock of reserves equal to $S_1$ tons. Deposit 2 has unit costs of $c_2$ and reserves equal to $S_2$. As long as the demand curve remains stationary, only the low-cost deposit will be exploited initially. Why? Suppose deposit 1 is the low-cost deposit. Its extraction costs are shown in Figure 3.11 as $c_1$. Extraction from deposit 1 commences at $T_1$, where the initial rent earned is the amount $ab$. Deposit 2 clearly cannot come on stream at $T_1$ because at price $p_1$, it will incur a substantial loss. Its extraction costs of $c_2$ greatly exceed the initial price.

How is this initial price set? Why isn't the initial price high enough to allow both deposits to operate? One way to see why the initial price is less than the extraction costs of the high-cost deposit is to work backward from the time when both deposits are exhausted—that is, we use the terminal condition. As long as the choke price $\bar{p}$ is greater than $c_2$, we know that physical exhaustion must occur. At $T$, $q(T) = 0$. The rent at $T$ would be equal to $\bar{p} - c_1$ for deposit 1 and $\bar{p} - c_2$ for deposit 2 if both extracted their last ton of ore at this point. Deposit 1's rent greatly exceeds that of

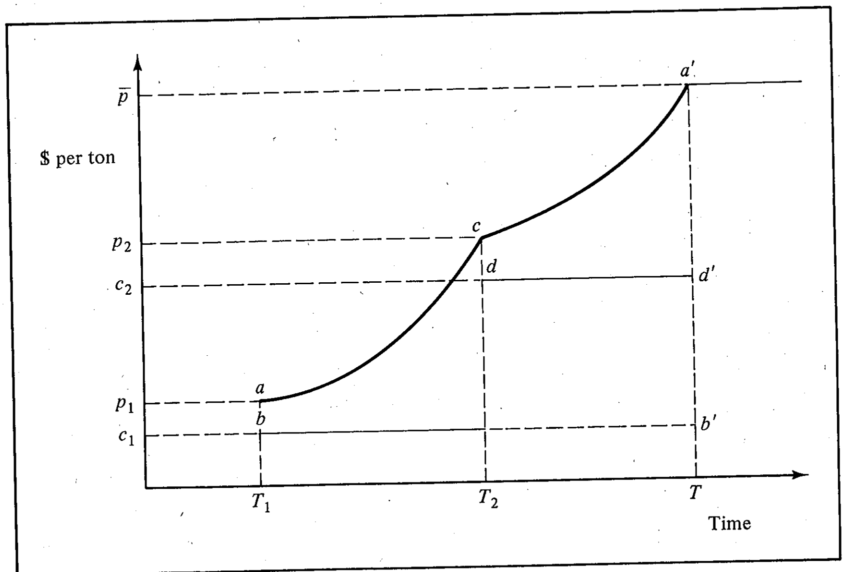



**Figure 3.11** The low-cost deposit is extracted between $T_1$ and $T_2$. Upon exhaustion, the second deposit is worked until it is exhausted at time $T$. $\bar{p}$ is the cost of the backstop. Rents rise over each phase at the rate of interest.



deposit 2. Can we then arrange extraction in each period prior to $T$ so that the flow condition is met for each deposit? The answer is "no" if each deposit operates simultaneously and "yes" if they operate sequentially.

Consider simultaneous extraction. The price in each period must be the same for each deposit if both are to sell any metal. There is then no way that the flow condition can be met for both firms. Working backward from $T$, the discounted rent cannot simultaneously be the same for each deposit with different extraction costs and the same market price. In particular, if a path such as $a'c$ is followed, rents to deposit 1 will not be falling fast enough from $T$. Only deposit 2 can then extract and satisfy the flow condition.

If there is sequential extraction, then for each deposit both the flow condition and terminal condition will be met, along with physical exhaustion. Working backward from $T$ where the choke price is reached, deposit 2 will be extracted over the interval $T_2$ to $T$. $T$ is defined by the choke price; $T_2$ is defined by the initial rent deposit 2 can earn to satisfy the flow condition for each period and exhaust at $T$. The initial rent for deposit 2, the amount $cd$, will be that which compounded at rate $r$ yields the terminal rent $a'd'$ as the ore body is depleted. $T_2$ is also the time when deposit 1 physically exhausts its reserves.

It will not pay the owners of deposit 1 to extract once the price has reached $p_2$, because they know that then deposit 2 can begin extraction. If both 1 and 2 were to extract simultaneously, the market price would not rise fast enough to allow 1 to satisfy the flow condition, as noted earlier. $T_2$ thus marks deposit 1's terminal time and $p_2$ its choke price. For deposit 1, the initial price, $p_1$, is what results from the output level chosen to obtain rent $ab$. Rent $ab$ is that amount which compounded at rate $r$ allows deposit 1 to exhaust its reserves as it reaches $p_2$. The scalloped price path shown in Figure 3.11 is the result.

We know that the scalloped price path must be continuous at the point where one deposit exhausts its reserves and the other begins extraction. In Figure 3.11, the point of the scallop, $T_2$, does not have any jump in the price upward or downward. To qualify as an optimal path, it must be the case that price does not jump at the transition between deposits. Consider what would happen if the price did jump. Suppose the price jumped up at $T_2$. The operator of deposit 1 would then decrease the extraction rate from the deposit immediately before time $T_2$ and extract this additional amount at $T_2$. The increase in the total supply at $T_2$ causes the price to fall. Production will continue to shift to $T_2$ until the jump in price is eliminated.

Similarly, if the price jumped down at $T_2$, ore from deposit 2 would be shifted to periods right before $T_2$. This would cause the price to fall prior to $T_2$ and rise at $T_2$, again eliminating the jump. As long as the extraction rate can be adjusted from one period to another, the transition from one deposit to another is smooth—there is no discontinuity in the price between one instant in time and the next.

Does this multideposit variation of the basic industry model have any real world applications? Nordhaus (1973) set out a numerical example of a multideposit model for the world energy market. The common output was a BTU (British thermal unit) of energy. The different "deposits" were distinct sources of energy—oil, gas, coal, uranium, and fusion as the backstop technology. The endogenous variables in his analysis, what the model determined, were the durations of the phases of exploitation of different energy sources and the price path. He took as given demand, costs, and "deposit" sizes. A linear programming approach was used to solve for an optimal extraction program. The analysis revealed that even before the OPEC price hikes of 1973, actual energy prices were slightly above optimal prices. He attributed this discrepancy to market imperfections, including noncompetitive behavior, and various forms of government regulation and taxation. Figure 3.11 illustrates the type of price path derived by Nordhaus.

## Setup Costs for the Mine and Industry

Clearing away overburden, building access roads, sinking shafts into the ground and pipes into the reservoir all represent infrastructure or setup costs—expenses that must be incurred before extraction commences. How will these setup costs affect the rate of extraction from the mine? What will happen to the industry extraction path when deposits have different setup costs? We turn now to these questions.

We first consider the effect of setup costs on the individual deposit. Two questions will be examined. How much physical capital or infrastructure should be installed in the mine or well, given that the size of the shaft or the diameter and pressure in the pipe constrain the flow of resource to the surface and hence the amount that can be sold? Once the physical capital is in place, what is the optimal extraction path? How does it differ from the path derived without considering capital requirements?

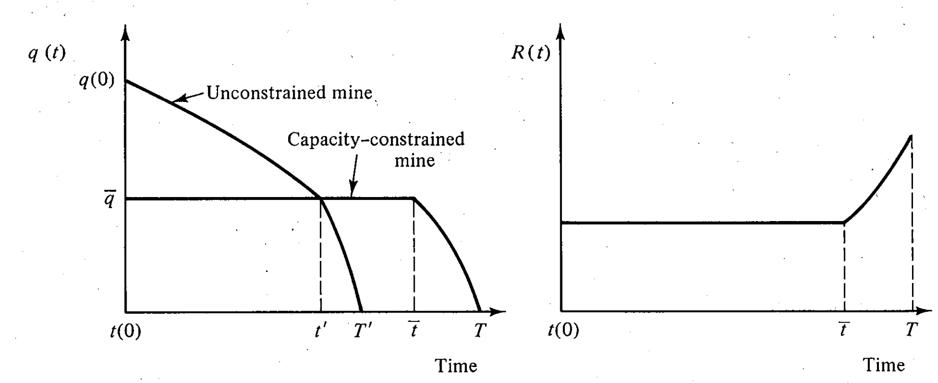
To determine the optimal size of a mine, the mine operator simply maximizes the difference between the contribution of infrastructure to the present value of mineral rents and the cost of the infrastructure. Let the capital be denoted by $K$. Then for capital of size $K$, the present value of the profits derived from the mine are $R(K)$. These profits will be the discounted value of the rent per period (revenues minus operating costs). If the amount of capital is increased, a larger flow of output, $q(t)$, could be extracted per period and the stock of reserves removed more quickly. A larger capital stock, however, increases capital costs, $C(K)$. The mine operator will then determine the value of $K$ that maximizes

$$R(K) - C(K) \tag{3.13}$$

If Equation (3.13) is differentiated with respect to $K$, the efficient condition for $K$ is obtained: $R_K - C_K = 0$. The change in the present value of the mine due to an incremental unit of $K$ added to the mine ($R_K$) must equal the marginal cost of adding that unit of capital ($C_K$).

Now that the optimal $K$ has been chosen, there is a maximum amount of ore (or oil) that can be removed from the mine or well at any point in time. The capital in place acts as a capacity constraint on the mine. To see the effect of the capital choice and resulting capacity constraint on the mine's extraction path, refer to Figure 3.12.[13] We show the path of output in (a) for both the constrained mine and a mine that does not have to install capital before extraction. The unconstrained mine in a Hotelling industry will extract its maximum output in the initial period, $q(0)$, then extract decreasing amounts each period thereafter until in the last period it produces, output goes to zero. The unconstrained mine exhausts its reserves at $T'$.






(a) Extraction paths for unconstrained and capacity–constrained mines

(b) Rent path for the constrained mine

**Figure 3.12** The effect of setup costs on the extraction and rent paths of the mine. Output and rents are constant over the time $t(0)$ to $\bar{t}$ of the mine's extraction period, for the capacity-constrained mine.

In general, the mine that must install capital and incur the setup costs will not choose an initial capital stock large enough to extract $q(0)$ because to do so would not maximize its rents. Why install a shaft large enough to remove $q(0)$ for only one period, and then have *excess capacity* over the remainder of the extraction period? Capital will be chosen such that a smaller amount of ore is extracted initially—$\bar{q}$ in Figure 3.12. Then the mine owner will produce at this maximum capacity level $\bar{q}$ for a period of time. The constant output in turn means that current (not discounted) resource price and rent (which now includes a shadow value for the capacity constraint) are constant, as shown in Figure 3.12(b).

How long will the mine produce at $\bar{q}$? The stock constraint and terminal condition again help us solve this problem. If the mine has uniform quality and constant costs, the owner will maximize profits by physically exhausting the reserves. At $T$, the end of the mine, output goes to zero. Typically, as is shown in the unconstrained case, the output does not jump to zero, but declines gradually. The same occurs in the constrained mine. After time $\bar{t}$, output diminishes from its constant level to hit zero at time $T$. If the stock of ore is the same in both the constrained and unconstrained mine, the only way the terminal condition can be met for the constrained mine is if it operates at full capacity over a period of time longer than the entire extraction period of the unconstrained mine. Thus, $\bar{t}$ exceeds $T'$. The area under the extraction path for both cases must be identical. The unconstrained mine will produce more than the constrained one over the interval $t(0)$ to $t'$, less thereafter, and exhaust at $T'$. The constrained mine will extract a constant amount over the interval $t(0)$ to $\bar{t}$, then extract decreasing amounts until it exhausts at $T$, where $T$ must exceed $T'$.

Now suppose different large deposits have different setup or capital costs. Each deposit will have a capital cost, $C^i(K)$, which we assume is incurred prior to exploitation of the ore. We now treat $C^i(K)$ as exogenous and *independent of the stock size or extraction costs.* This is reasonable if we are thinking of building roads, but less so for reserves of different sizes and characteristics. The assumption simplifies the analysis and illustrates the new complexities these costs can introduce.

Two new phenomena emerge. First, the industry price path has jumps down in the price at the transition between deposits. Second, the optimal plan differs from the competitive solution. The latter phenomenon occurs because setup costs introduce increasing returns to scale into the analysis.[14]

Consider how the jump in the price arises. Let us return to the two-deposit example of the preceding discussion, but add a small setup cost that occurs at the transition from deposit 1 to deposit 2. Figure 3.13 illustrates the result. As noted earlier, $T_1$ to $T_2$ is the extraction period of the first deposit, while $T_2$ to $T$ represents the time of extraction for the second deposit. But if there is a setup cost to obtain ore from deposit 2, the value of extraction from the industry beyond the exhaustion of the first deposit's reserves is diminished. To counteract this new lower value beyond $T_2$, one can get higher rents from the first deposit by depleting it over a longer interval. This is in part because the farther into the future one can defer paying the setup costs, the lower the present value of these costs.

To postpone reaching the second deposit, less must be extracted in each period from the first deposit than was the case with no setup costs. The price of the mineral will therefore be higher in every period up to $T_2$, and the interval $T_1$ to $T_2$ must lengthen. At $T_2$, the extraction plan for the second deposit proceeds as before. Given this lengthening of the interval $T_1$ to $T_2$, the price must *jump down* at the switch from deposit 1 to deposit 2. The higher the setup costs, the greater the jump down in the price. The extraction rate from deposit 1 will be lower the larger the setup costs to reduce the present value of these costs.

The determination of the optimal path becomes much more complex if all deposits have setup costs. Prices do not link up as they did in Figure 3.11. More important, the competitive market equilibrium will no longer yield the optimal extraction path. Consider the special case of identical deposits, each with a setup cost.

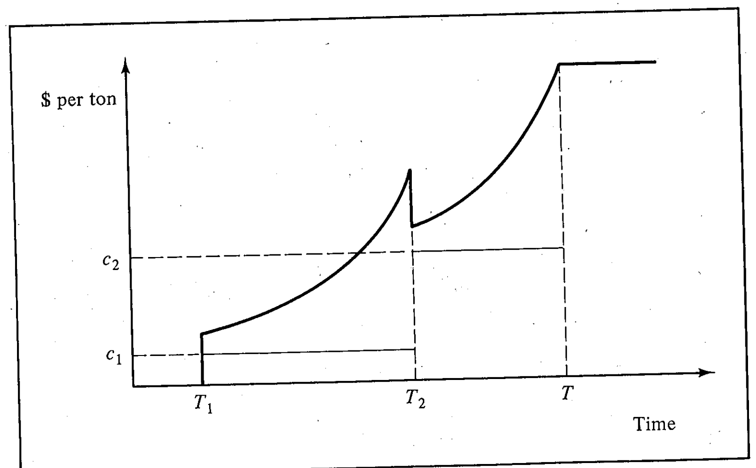



**Figure 3.13** To deposits of different quality, a positive setup cost of "opening" the second deposit is added. This results in an optimal extraction path with a jump down in price at $T_2$, the date deposit 1 is exhausted.



From a social welfare standpoint, it is irrelevant which deposit is exploited first. However, in the firms' optimal plans, the discounted rent net of setup costs will be different for deposits entering at different times in the extraction sequence. The first deposit will get less rent than the second, the second less than the third, and so on, because early producers "pay" a higher setup cost in present value terms. The setup costs cannot be fully offset by higher initial prices when all firms must bear the costs. In a decentralized market, deposit owners will prefer to go at the end of the sequence, and there is no market mechanism to decide which deposit goes into which slot in the sequence.[15]

The reason for the breakdown of the market mechanism in the presence of setup costs is that these costs represent a form of increasing returns. The larger the deposit over which the setup costs can be spread (or the longer the time period over which they can be spread), the higher the rents to the mine owner. Small setup costs or slight increasing returns to scale cause less deviation from the familiar paths than large setup costs. Any form of increasing returns to scale leads to malfunctioning of the market as a mechanism for allocating productive resources optimally. The malfunction shows up in an unusual manner in the nonrenewable resource case.

One final point: Throughout this chapter, we have identified a mineral deposit with a mine owner or operator. This is convenient in the case of setup costs because it implies that a single coordinator incurs the costs. In a situation of many owners of small claims on a single deposit, it is difficult to see how the sharing of setup costs would be arranged. Also, for deposits with single owners, the individual last in a sequence will be induced to price as a monopolist, since all competitors will have exhausted their stocks. This again breaks down the socially optimal path. We turn to monopoly in the next chapter.

## SUMMARY

1. Nonrenewable resources differ from reproducible goods because they have a fixed stock of reserves that, once removed, cannot be replaced. A unit of ore removed today means that less in total is available for extraction tomorrow.
2. The economic theory of extraction explains the flow of production over time and how quickly the resource stock is exhausted.
3. The finite stock of a nonrenewable resource alters the condition for efficient production: Marginal revenue *(MR)* equals marginal cost *(MC)* in three ways: (a) $MR = MC +$ resource rent; (b) the present value of resource rent must be constant for each period the mine operates; (c) the total amount of the resource extracted over time cannot exceed the total stock of reserves. Conditions (a) and (b) yield the flow condition, while (c) is the stock constraint.
4. Extraction from a mine facing a constant price, a positive discount rate, and extraction costs that do not increase as the stock of ore is depleted decreases in each period the ore is removed. This is physical depletion.
5. For a many-period model, the terminal condition, marginal rent equals average rent, determines the time horizon over which the mine operates.
6. Different ore qualities within the mine require the price of the mineral to rise over time for extraction to occur. If price does not rise sufficiently, the

mine will cease operating before the reserves are exhausted. This is economic depletion.
7. Extraction from a mineral industry facing a negatively sloped demand curve and constant costs of extraction will be socially optimal if the flow condition and stock constraint are satisfied. Resource rent per unit extracted will then rise continuously at the rate of interest. This is known as Hotelling's rule, and it is met if all mine operators have perfect foresight, with industry output declining over time.
8. A nonrenewable resource with a backstop technology will be extracted until the price of the resource reaches the choke price, that price at which the backstop becomes profitable to produce.
9. The following comparative static results were derived: (a) an increase in extraction costs tilts production to the future and increases the time to depletion of an ore body; (b) an increase in the interest rate tilts production to the présent and reduces the time to depletion of an ore body; (c) taxes on mineral rent do not alter the extraction path of an existing mine or well, but they will affect the incentive to find new deposits; (d) a royalty on mineral production tilts production to the future and increases the time to depletion.
10. Low-cost deposits will be exploited before high-cost deposits, leading to a scalloped but continuous price path.
11. Setup costs within the mine can lead to periods of constant rates of extraction and constant prices.
12. Setup costs for the industry can lead to jumps (discontinuities) in the optimal price path.

## DISCUSSION QUESTIONS

*1.* If the interest (discount) rate is zero, what is the value of resource rent over the extraction profile of the mine?
*2.* Using Gray's model, derive for a two-period case the extraction path of a mine's output assuming:
   *a.* Extraction costs (average and marginal costs) are linear and upward sloping.
   *b.* The market price of the mineral rises; the market price falls.
*3.* Suppose a mine has two different ore qualities in its stock of reserves. Call them block A and block B. How would the mine owner efficiently extract the total stock if the costs of extraction are constant per unit within each block, but differ between blocks? Use the Gray model.
*4.* In the basic Hotelling model of the industry, extraction cost per ton was constant. In order to reflect, say, diminishing returns to the extraction facilities in the industry, let cost per ton rise with the amount extracted in a period in the industry (as in the simple L. C. Gray model of the mine). Compare two programs of quantities extracted: one with constant costs and one with extraction cost per ton rising linearly with quantity extracted in a period.
*5.* Explain and show diagrammatically that a price path which does not reach the choke price in the basic industry model is nonoptimal and will not occur under perfect foresight.
*6.* In the basic Hotelling model of the industry, with constant unit extraction costs and a negatively sloped industry demand curve, technological progress in extraction can be approximated by a decline in the value of the constant extraction costs period by period. Outline how the program of quantities extracted with a 2 percent decline in unit extraction costs period by period compares with the program of quantities extracted when unit extraction costs remain constant.
*7.* Derive the effect on a mineral industry's output and price path if, at some point along an optimal path, the costs of extraction rise and:
   *a.* The increase in costs is fully anticipated (foreseen) by the industry.
   *b.* The cost increase is completely unanticipated.
*8.* What are the effects on a mineral industry's output and price path of a fully anticipated:
   *a.* Increase in the total stock of ore reserves.
   *b.* Fall in the choke price (cost of backstop technology).
   *c.* Technological change that decreases the cost of extraction over time.
   *d.* Rightward shift in the demand curve.
*9.* How would the price path for a competitive industry differ if it faced an isoelastic rather than a linear demand curve?
*10.* Explain why the higher the setup costs a mine faces, the longer it will produce at a constant output rate and the longer the life of the mine.



## NOTES

1. An important exception to the finite stock of minerals are seabed nodules. Minerals such as nickel, copper, manganese, and molybdenum have been found on the ocean floor and might be growing over a time period much shorter than the millions of years required to produce hardrock minerals, oil, and gas on land. Although the precise way in which these seabed nodules are formed is not yet clear, there is some indication that they may be cropped periodically and will re-form.
2. We do not consider here any of the capital costs associated with thé development of the mine. There are no shafts to dig, pipes to install, mills to build. A later section of this chapter considers these capital or setup costs explicitly.
3. The cost curves illustrated in Figure 3.1 were chosen to show simply how the efficient extraction path is derived for the mine. We could use linear or strictly convex cost curves instead, but the discussion of the conditions for efficient extraction would not be as intuitive. If the U-shaped cost curves are used to examine mineral extraction at the industry level, there can be a problem because a competitive market equilibrium may not exist. See the industry section of this chapter and the discussion by Eswaran, Lewis, and Heaps (1983).
4. Condition 2 can also be written as $R(0)(1+r) = R(T)$ by dividing through by $(1+r)$. We will use both forms of the condition.
5. This is L. C. Gray's many-period model in which the length of time for exhausting the mine must be arrived at.
6. We assume there are no market imperfections of any kind in this analysis, including externalities such as pollution associated with mineral extraction, and that the market interest or discount rate is the social rate of discount as well.
7. Actual mineral markets are frequently characterized by disequilibrium caused by a variety of factors, especially imperfect competition and government regulation. We turn to these issues later.
8. Assuming perfect competition, if the substitute is a reproducible good, the choke price will equal the marginal cost of producing the good. If the substitute is another nonrenewable resource, the choke price is its marginal cost plus rent. See Figure 3.11 for an illustration of the latter case.
9. A demand curve of the form $q = Ap^{\epsilon}$ where $\epsilon$ is the elasticity of demand will be asymptotic to the price and quantity axes for $\epsilon < 0$. Thus, price can approach infinity and quantity

will tend to zero. For $-1 < \epsilon < 0$, the demand curve is inelastic and revenue, $pq$, will decline as price rises. There is no intercept on the price axis for this demand curve and hence no choke price.

10. In Chapter 5 we consider uncertainty in the introduction of the backstop technology.
11. Note that the last batch extracted from the stock will be $q_z$ tons, as indicated on Figure 3.6. At this quantity, the rent on the marginal ton will be less than average rent on the tons in the batch. Should not more resource be brought into the end of the extraction plan from the front? The answer is "no." If a few batches of $q_z$ tons at the end of the plan were sold at \$Z, then the present value of rent on the marginal ton would be falling, because price is constant at \$Z. The present value of rent per ton on these last batches would be less than on tons extracted early. This violates the principle of identical tons receiving identical rents. In the optimal plan, we are left with average rent above marginal rent for tons in the final period of extraction of the exhaustible resource.
12. There are, of course, many different types of taxes levied on mining industries. In this section we assume the tax rate is constant over time, but in many real-world examples, the rate varies considerably over time. See Dasgupta, Heal, and Stiglitz (1980) for a discussion of the effects of different types of taxes and tax rates on the optimal extraction path.
13. The discussion in this section is based on Campbell (1980). See Olewiler (1980) for a discussion of the possible nonexistence of a competitive industry equilibrium with the capacity constraint. It is possible that the capacity constraint need never bind. If it does bind at all, it need do so only in the initial period of extraction.
14. Whenever average cost curves for a firm slope down, there are some increasing returns to scale, which are a type of nonconvexity. The Gray model was presented with declining average costs. At the level of the mine, this presents no difficulties. At the industry level, however, we can expect a market failure to occur, as will be discussed.
15. This problem may be less severe in practice, because mines are discovered at different points in time. The total stock of minerals in the world is not known at any time. Development of the mine and incursion of the setup costs frequently takes place when market conditions warrant (when demand exceeds supply). In cases where mines do "wait" to go into production to minimize setup costs, the surplus rents to these mine owners at the end of the sequence are likely to be capitalized into the value of the mine or bid away by workers as higher wages and fringe benefits.

Reading 1

## ECONOMICS AND GEOLOGY: ARE NONRENEWABLE RESOURCES FINITE?

What is a nonrenewable resource? We speak of it as a finite stock of some physical amount of metal, oil, gas, or other material. Is it really fixed? Why have we observed measured stocks of many minerals increasing rather than decreasing over time? What we are trying to determine is the total supply of a nonrenewable resource—the maximum amount that can be extracted.

In nineteenth-century England, economists debated whether coal consumption should continue at present levels or be saved for the future because of concern over dwindling reserves. In the previous century, the stock of wood used to generate heat had dwindled considerably. Was this a response to the abundant supplies of the new substitute, coal? Or was the coal era brought on because the stock of wood was approaching exhaustion? In 1874, a geologist from Pennsylvania State University calculated that the United States had only enough petroleum to keep kerosene lamps burning for another four years! In the 1970s, many



countries considered implementing gasoline rationing to deal with the "oil crisis." But the world has not yet run out of a nonrenewable resource, and some scientists argue that we never will: "We will never physically "run out" of a mineral, because the sheer limits of the amount in place in the ground are far beyond the likely economic limits of its utilization" (Zwartendyk, 1972, pp. 8–9).

But even if we don't worry about ultimate exhaustion, measures of resource stocks are important for private and public decision-making. There are no absolute or unique measures of resource stocks. Economists differ from geologists, who differ from mining engineers in their calculations. Part of the reason for this is that people use different terminology. Are *proved* reserves the same as *measured, probable* the same as *indicated?* But even if the terminology could be standardized, the basic problem is that a mineral stock is not a well-defined or even a meaningful concept. To see the problems, consider two extreme views of what constitutes the stock of a mineral.

For the narrowest definition, the known stock of a mineral is that which can be profitably extracted given current technology, extraction costs, prices, and political factors. Once a deposit is found, test holes are drilled into the rock formations and then, by a variety of techniques, the deposit is delineated. Calculations of *total reserves* within the deposit are made based on these sample drill holes. The ore that is counted in the calculation is that *which at the time* the calculation is made is believed to be economically viable—that is, profitable to extract over the foreseeable future.

The difficulties with this technique are fairly obvious. A sample can give a very biased picture of the total. There are many examples of mines started up on the basis of reserve estimates that were not representative of the deposit. Sometimes this is good news—more is there than anticipated—but often the opposite is true: The vein disappears, the ore grades tail off too rapidly to permit extraction, and so on.

Another difficulty is that geologists and engineers do not have perfect foresight about mineral prices, costs, and technology, all of which determine whether a deposit is viable or not. World gold reserves are very small when viewed from a gold price of \$35 per ounce, but much less so at \$500 per ounce. The only certain measure of what a deposit contains is made after all the ore is removed and the mine shuts down. And even here there are difficulties because depletion may result from economic forces (which are not constant), not physical exhaustion.

Finally, what is counted as reserves in one country may not be in another. A 400-million-ton porphyry copper deposit in Canada with an average grade of 0.25 percent copper can be counted as a viable reserve. The same deposit would not be counted as part of Chile's copper reserves unless its ore grade was at least twice as much. Does Chile therefore have fewer copper reserves than Canada?

We have been discussing *known* deposits in the narrow definition of stocks. A very important source of the growth in mineral reserve estimates over the past fifty years has been the discovery of *new* deposits. Although geologists may have a notion that additional deposits should exist, they still have to be found if a reasonable estimate of the mineral stock is to be made.

Suppose we want to know the maximum potential stock of a mineral. We go then to the opposite extreme—the *resource base.* This is what geologists call the concentration of minerals in the earth. It is a purely geologic concept, without regard to technological feasibility or economic viability. As Brooks (1976, p. 148) explains: "Measurements of the resource base are stable, for neither individual discoveries nor depletion changes the volume significantly." However, there are difficulties even with this concept. It is fairly useful for fluids such as petroleum, because they are distinct from the rocks that surround them and are found only to some maximum depth in the earth. But for many metallic minerals, the resource base is not so easy to define. Metals in minute concentrations are in a large part of the earth and to an unknown depth. At what point do we stop counting? A number of cutoff points have been suggested, but no system has been agreed upon.

Alternative measures are to count minerals that can be extracted without crossing an "energy barrier." This implies that minerals will be counted as long as their extraction or separation from surrounding material does not use up large amounts of energy (where "large" is not defined). The appeal of the resource base is that it is a physical measure of the maximum stock of any mineral. The difficulty with the resource base as a measure of mineral stocks is that it does not indicate whether these potential supplies will ever become actual. Supply is "actualized" only when with given geological knowledge and information about extraction techniques, there is "willingness to pay" for the mineral.

An economic measure of a mineral stock lies between the narrow definition of reserve estimates based on drill hole samples and the broad measure of the resource base. The economist is concerned about not only which deposits are feasible today, but given that technology, costs, and prices are rarely constant over time, how much of the resource base will become viable reserves over time. The economist tries to draw a supply picture that incorporates effects of new discoveries, exhaustion of old deposits, and changes in prices, costs, and technologies, using statistical techniques when possible to determine supply elasticities, shifts in supply curves, and so on. The crucial thing to the economist is that the stock of reserves *will change* over time. Some reserves will come from the unanticipated discovery of new deposits; some will come from the movement of previously uneconomic resources into economic reserves due to unanticipated increases in prices, decreases in costs, and technological change.

An economic definition of reserves is illustrated in Figure 3.14. On the vertical axis, we have the expected discounted value of mineral rents (price minus unit cost). This is a slightly broader measure of mineral rents than contained in the chapter because it is an expected value. It takes into account the probability of discovering new ore bodies (which will affect the costs), the likely path of prices over time, technological change, and so on. Every ton of ore or unit of oil that can be extracted and earn a nonnegative rent will be counted as a reserve, whether it has been discovered or not. Due to differences in ore grade, there will be relatively few units of a mineral that generate high rents. Ores of lower grade (higher costs) will yield lower rents. Thus the relationship between discounted rents and mineral reserves is likely to be downward-sloping (it need not be smooth as shown, but may have wiggles or bumps). What then is the mineral stock?

The point at which this negatively sloped curve crosses the quantity axis will determine the stock of reserves. That is, where the present value of rents equals zero, the marginal ton that can be extracted is defined. Will this be a fixed stock forever? The answer depends on how good our expectations are. If we do have perfect foresight about prices, technologies, discoveries, and so on, there will be a unique relationship between rent and reserves. What is more likely is that we will be surprised—either pleasantly, in which case the reserve line shifts out and aggregate reserves rise, or unfortunately, when anticipated cost savings, discoveries, or price movements do not materialize and the curve shifts in. We expect that the reserve estimate will not be a fixed stock, but will change over time and is thus more appropriately called a flow. Figure 3.14 illustrates one hypothetical reserve estimate. (These uncertainties are examined in more detail in Chapter 5.)

So is there a unique measure of the stock of each mineral? In physical terms, "yes," if one is willing to specify some lower bound of concentration in the earth. In practical terms and for policy analysis, we are really more concerned with the *supply* of the resource potentially available for extraction. Although this supply can be fixed at points in time and extraction decisions based on a stock constraint, it is really not a stock at all, but the flow over time of resources into reserves. And as a geologist has argued: "As long as this flow can be maintained in a workable fashion, we need not worry about the absolute magnitude of the shelf inventory" (Zwartendyk, 1972, p. 11).



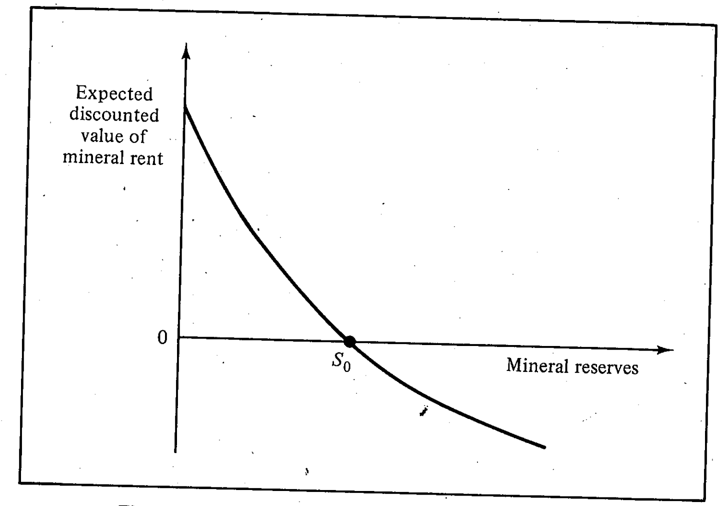



**Figure 3.14** Mineral reserves as a stock and flow.

Reading 2

## MINERAL INDUSTRIES OVER TIME? A "TEST" OF THE THEORY

Economic models have many functions. They enable us to understand real-world phenomena by breaking them down into manageable abstractions. Models also give rise to testable predictions about the economic activities they describe. We illustrate the use of theory to examine real-world phenomena in an examination of mineral prices over time. In a recent paper, Margaret Slade (1982) attempted to explain the price paths of a number of mineral resources over a long period of time. Once adjusted for inflation, the prices of these minerals do not look like our illustrations in this chapter. Rather, there are periods of rising and periods of falling prices, along with some periods where the price changes very little. What Slade argues is that it is changes in the cost function over time which can explain the price path of many minerals.

There are two components of the cost function, she asserts, which work in opposite directions as the industry's stocks of reserves are depleted over time. As extraction proceeds, mines typically must extract ore of increasingly lower grade. We argued that ore bodies of high quality will be mined before ore bodies of low quality. Slade finds evidence of this for many minerals. In copper deposits, for example, the average ore grade mined in the early 1900s was 5 percent. Now it is closer to 0.7 percent (Slade, p. 126). Following our model, we would expect that the decline in ore grade increases the average and marginal costs of extraction. Rising costs will then lead to rising mineral prices (see Figure 3.8).

Offsetting these cost increases due to lower ore grade is technological change. According to Slade (p. 126): "Technological developments in the early part of the century, particularly the advent of large earth-moving equipment, which made possible the strip mining of extremely low-grade ore bodies, and the discovery of froth flotation, which made concentration of low-grade sulfide ores very economical" led to a fall in many mineral prices over the early part of the twentieth century because of the decline in extraction costs brought about by these technological changes. However, the rate of technological change in many mineral extraction processes has slowed considerably in the second half of this century. Technological change appears to be less able over time to offset the cost increases due to declining ore grades. If so, we would expect costs and mineral prices to rise.

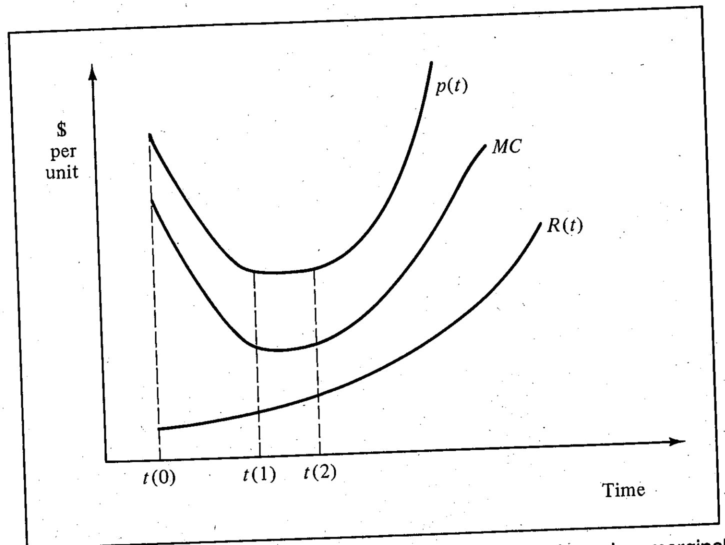



**Figure 3.15** Marginal cost, prices, and mineral rent over time when marginal cost depends on the rate of technical change and ore grade. Prices initially fall because the rate of technological change offsets ore grade decline and *MC* falls. Eventually technological change can no longer offset cost increases due to falling ore grades, and the price path slopes up. Rents over time, *R*(*t*), are everywhere increasing to satisfy Hotelling's rule.

Slade modifies a Hotelling-type model to incorporate these assumptions about ore grade and technological advances. The model yields price, rent, and cost curves over time as shown in Figure 3.15. The path illustrated in Figure 3.15 is a stylized representation of three different regions of the price path. Over the period $t(0)$ to $t(1)$, prices are falling because the rate of technological change determines the rate of decline in ore grade. Marginal (and average) extraction costs fall. From $t(1)$ to $t(2)$, prices are stable because the two cost terms cancel one another. After $t(2)$, the rate of technological change is no longer high enough to offset the ore grade decline and the marginal cost curve increases, leading a rising price path. Notice that the mineral rent in current dollars is always increasing, so that Hotelling's rule (the flow condition) is met.

The model is then "tested" against actual prices of 12 mineral commodities for the period 1870 (or since the year of earliest available data for some minerals) to approximately 1978. Two price equations are estimated for each mineral—one where the price is a simple linear function of time, the other where price is a quadratic function of time. If the cost effects described above are a good description of mineral extraction over time, the quadratic function should fit the time series of prices better than the linear function. The linear function does not allow for changes in costs over time and therefore would be inconsistent with the cost assumption made in the theoretical model. Slade found that no discernible trend could be seen with the linear function. For some minerals, prices rose over time; in others, they fell; and in some, price was virtually constant. In the quadratic case, however, for all 12 commodities examined, the linear term was negative while the quadratic term was positive. These time coefficients were highly significant statistically for virtually every commodity. The mineral price paths do appear to be U-shaped as predicted by the model. The quadratic function is thus a better general description of the data than the linear function.

More specifically, Slade found that for every mineral, price had passed the minimum point on the U-shaped curve by 1978. There were differences among the minerals in the extent

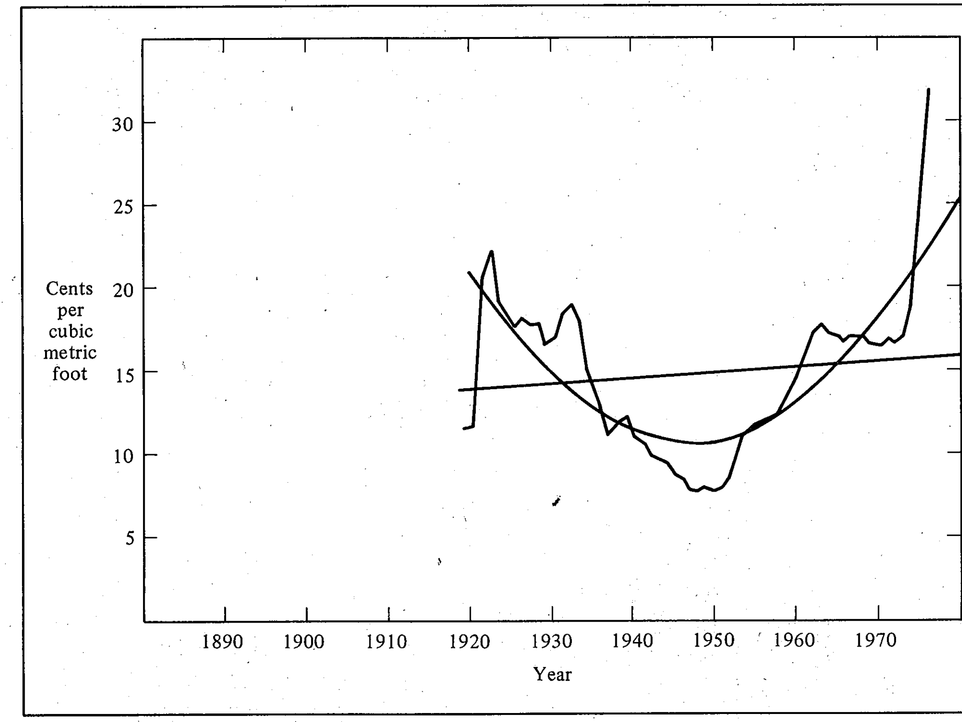



**Figure 3.16** History of deflated prices and fitted linear and quadratic trends for natural gas. (*Source:* M. Slade (1982), p. 130.)

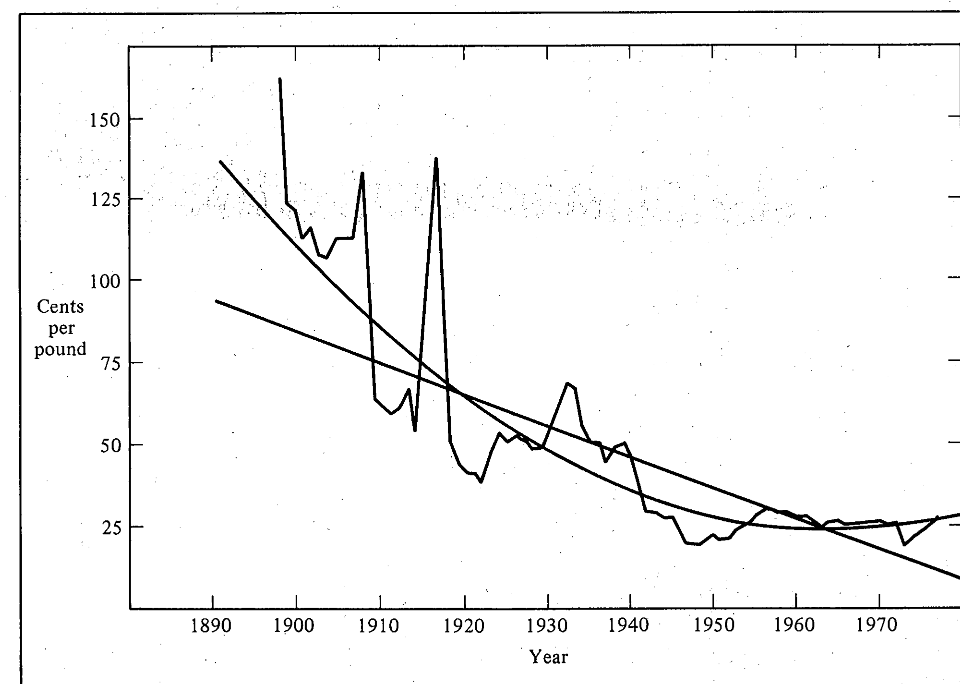



**Figure 3.17** History of deflated prices and fitted linear and quadratic trends for aluminum. (*Source:* M. Slade (1982), p. 131.)

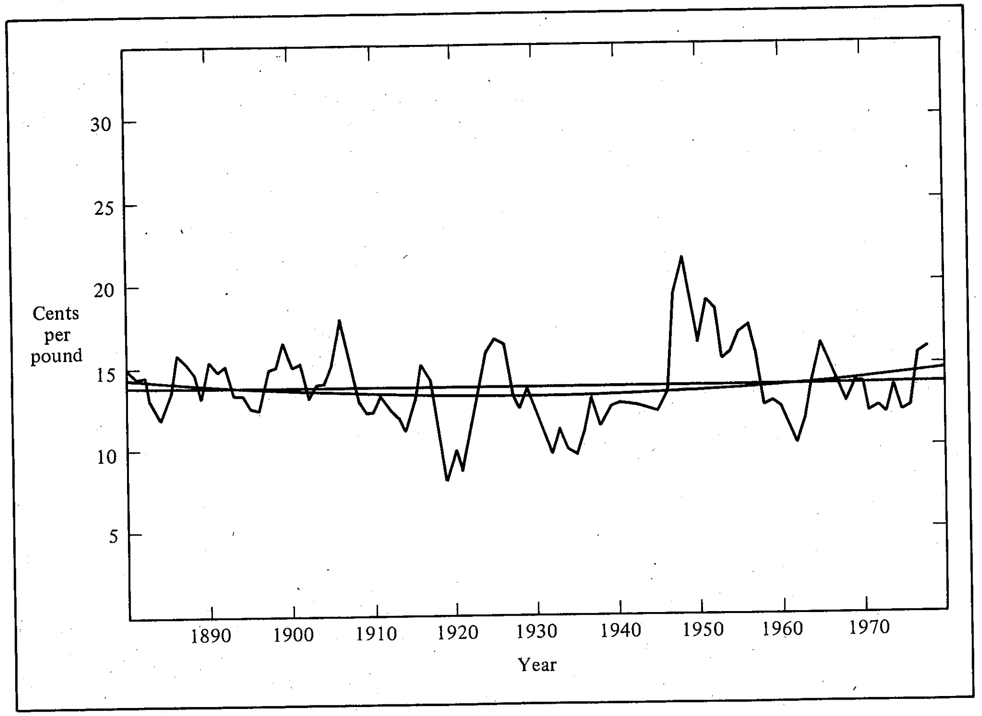



**Figure 3.18** History of deflated prices and fitted linear and quadratic trends for lead. (*Source:* M. Slade (1982), p. 133.)

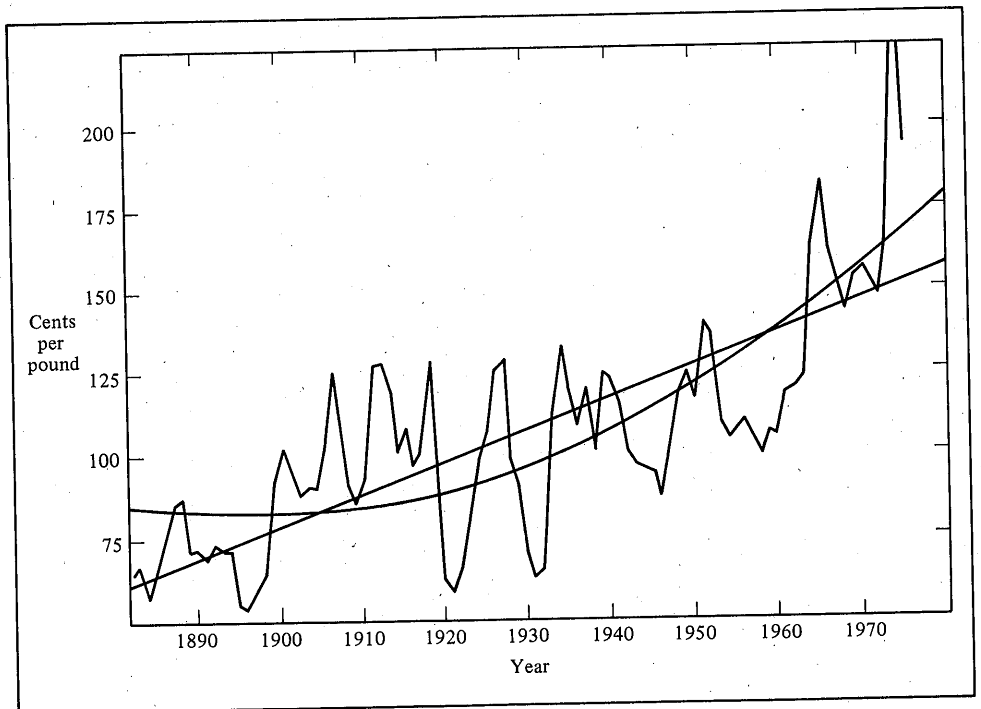



**Figure 3.19** History of deflated prices and fitted linear and quadratic trends for tin. (*Source:* M. Slade (1982), p. 134.)





of the U shape and point in time when the price began to rise, which Slade attributes to the specific characteristics of each commodity. For copper, iron, nickel, silver and natural gas, a pronounced U-shaped price path was estimated. The path for natural gas is illustrated in Figure 3.16. There are three other variations. In the case of aluminum, shown in Figure 3.17, the price path is generally falling over most of the period examined. Slade attributes this to high growth rates in aluminum consumption, combined with technological advances and economies of scale. Lead and zinc, on the other hand, are metals with relatively stable demand over time and technological changes that have just offset ore grade declines. Given this information, their predicted price paths would be relatively constant, and as seen in Figure 3.18, the quadratic function fitted to lead shows very little curvature (zinc is similar). Tin, a metal that has been in use for centuries, is characterized by steadily declining consumption rates and a substantial decline in ore grade. It is not suitable for *froth flotation* and thus has been unable to benefit from that technological change. As Figure 3.19 shows, its price path, while still having a small curvature, is basically upward-sloping.

The analysis shows that the theoretical model which predicts U-shaped price paths fits the mineral data very well. When detailed information is available about the resource's consumption patterns, ore grade decline, and ability to incorporate technological changes into the mining processes, the empirical analysis will show more precisely how the general model adapts to fit these cases. The model is simple and obviously does not capture all real-world complexities. However, when "tested" against real-world observations, it performs well and shows the value of using a theoretical model to help determine empirical relationships.